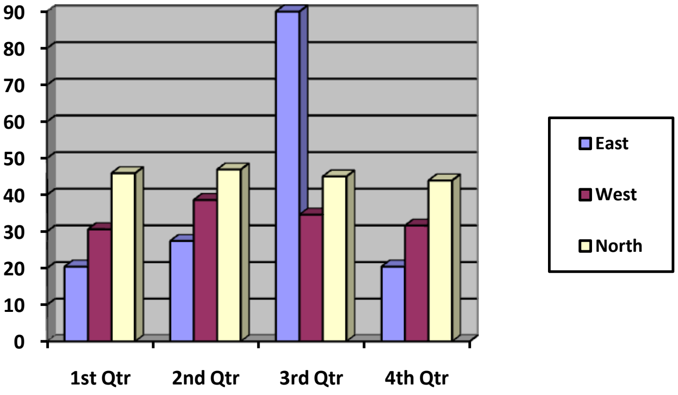
UNESCO-NIGERIA TECHNICAL &
VOCATIONAL EDUCATION
REVITALISATION PROJECT-PHASE II

NATIONAL DIPLOMA IN STATISTICS



DESCRIPTIVE STATISTICS 1

COURSE CODE: STA 111

YEAR I- SE MESTER I

THEORY/

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Week One

***General objective*:** Understand the nature of statistical data, their types and uses

***Specific goal*:** The topic is designed to enable students to acquire a basic Knowledge of definition of statistics

1.0 Definition of statistics

Statistics can be defined as a scientific method of collecting, organizing, summarizing, presenting and analyzing data as well as making valid conclusion based on the analysis carried out.

Statistics is a scientific method which constitutes a useful and quite often indispensable tool for the natural, Applied and social workers. The methods of statistics are useful in an over-widening range of human Endeavour, in fact any field of thought in which numerical data exist. Nowadays it is difficult to think of any field of study that statistics is not being applied, in particular at the higher level. Thus, statistical method is the only acceptable.

Or

Statistics is a scientific method concerned with the collection, computation, comparison, analysis and interpretation of number. These numbers are quite referred to as data. However, statistical mean more than a collection of numbers.

1.1 Importance/ uses of statistics.

Statistics involve manipulating and interpreting numbers. The numbers are intended to represent information about the subject to be investigated. The science of statistics deals with information gathering, condensation and presentation of such information in a compact form, study and measurement of variation and of relation between two or more similar or identical phenomena. It also involves estimation of the characteristics of a population from a sample, designing of experiments and surveys and testing of hypothesis about populations.

Statistics is concerned with analysis of information collected by a process of sampling in which variability is likely to occur in one or more outcomes.

Statistics can be applied in any field in which there is extensive numerical data. Examples include engineering, sciences, medicine, accounting, business administration and public administration. Some major areas where statistics is widely used are discussed below.

(a) *Industry*:- Making decision in the face of uncertainties is a unique problem faced by businessmen and industrialist. Analysis of history data enables the businessman to prepare well in advance for the uncertainties of the future. Statistics has been applied in market and product research, feasibility studies, investment policies, quality control of manufactured products selection of personnel, the design of experiments, economic forecasting, auditing and several others.

(b) *Biological Science*: - Statistics is used in the analysis of yield of varieties of crops in different environmental conditions using different fertilizers. Animal response to different diets in different conditions could also be studied statistically to ensure optimum application of resources. Recent advancement in medicine and public health has been greatly enhanced by statistical principles.

(c) *Physical Science*: - Statistical metrology has been used to aid findings in astronomy, chemistry, geology, meteorology, and oil explorations. Samples of mineral resources discovered at a particular environment are taken to examine its essential and natural features before a decision is made on likely investment on its exploration and exploitation. Laboratory experiments are conducted using statistical principles.

(d) *Government*: - A large volume of data is collected by government at all levels on a continuous basis to enhance effective decision making. Government requires an up-to-date knowledge of expenditure pattern, revenue, estimates, human population, health, defense and internal issues. Government is the most important user and producer of statistical data.

1.2 Types of statistical data

There are basically two main types of statistical data.
These are

(i) The primary data, and
(ii) The secondary data.

The primary data

As the name implies, this is a type of data whereby we obtain information on the topic of interest at first hand.

When the researcher decides to obtain statistical information by going to the origin of the source, we say that such data are primary data. This happens when there is no existing reliable information on the topic of interest.

The first hand collection of statistical data is one of the most difficult and important tasks a statistician would carry out. The acceptance of and reliability of the data so called will depend on the method employed, how timely they were collected, and the caliber of people employed for the exercise.

Advantages

The investigator has confidence in the data collected.

The investigator appreciates the problems involved in data collection since he is involved at every stage.

The report of such a survey is usually comprehensive.

Definition of terms and units are usually included.

It normally includes a copy of schedule use to collect the data.

Disadvantages

The method is time consuming.

It is very expensive.

It requires considerable manpower.

Sometimes the data may be obsolete at the time of publication.

The secondary data

Sometimes statistical data may be obtained from existing published or unpublished sources, such as statistical division in various ministries, banks, insurance companies, print media, and research institutions. In all these areas data are collected and kept as part of the routine jobs. There may be no particular importance attached to the data collected. Thus, the figure on vehicle license renewals and new registration of vehicles can first be obtained from the Board of Internal Revenue through their daily records. The investigator interested in studying the type of new vehicles brought into the country for a particular year will start with the data from the custom department or Board of internal revenue.

Advantages

They are cheap to collect.

Data collection is less time consuming as compared to primary source.

The data are easily available.

Disadvantages

It could be misused, misrepresented or misinterpreted.

Some data may not be easily obtained because of official protocol.

Then information may not conform to the investigator's needs.

It may not be possible to determine the precision and reliability of the data, because the method used to collect the data is usually not known.

It may contain mistakes due to errors in transcription from the primary source.

1.3 Uses of statistical data.

The following explain uses of statistical data.

(i) Statistics summarizes a great bulk of numerical data constructing out of them source representative qualities such as mean, standard deviation, variance and coefficient of variation.

(ii) It permits reasonable deductions and enables us to draw general conclusions under certain conditions

(iii) Planning is absorbed without statistics. Statistics enables us to plan the future based on analysis of historical data.

(iv) Statistics reveal the nature and pattern of the variations of a phenomenon through numerical measurement.

(v) It makes data representation easy and clear.

1.4 Definition of quantitative random variable

A quantitative random variable is that which could be expressed in numerical terms. They are of two types: Discrete and continuous.

Discrete random variable

These are random variables which can assume certain fixed whole number values. They are values obtained when a counting process is conducted. Examples include the number of cars in a car park, the number of students in a class.

The possible values the random variable can assume are 0,1,2,3,4, e.t.c.

Continuous random variable

This types of random variable assumes an infinite number of values in between any two points or a given range.

Continuous random variables are often associated with measuring device. The weight, length, height and volume of object are continuous random variables. Other examples include the time between the breakdown of computer system, the length of screws produced in a factory and number of defective items in a production run. In these cases, the numerical values of specific case is a variable which is randomly determined, and measured on a continuous scale. It should be noted that any numerical value is possible including fractions or decimals.

1.5 Types of measurement

There are four (4) types of data. Namely: - nominal, ordinal, interval and ratio.

Nominal data

This represents the most primitive, the most unrestricted assignment of numerals, in fact, the numerals is used only as labels and thus words or letters would serve as well.

The nominal scale has no direction and is applicable to numerals or letters derived from qualitative data. It is merely a classification of items and has no other properties.

For example, the people of Nigeria can be classified into ethic groupings such as the Ibos, the Yoruba's, the Hausas, the Ibibio etc without necessarily inferring that one ethnic group is superior to the other.

Ordinal data

The ordinal scale has magnitude, hence is a step more developed than the nominal scale. It has the structure of order- preserving group. Items are placed in order of magnitude. In fact, it is a group which includes transformation by all monotonic increasing function.

For example, if Bassey is taller than Obi and Obi is taller than Akpam, the rank in terms of tallness is Bassey first, Obi second and Akpam third. This scale merely tells us the order, but not specific magnitude of the differences in height between Akpam, Obi and Bassey.

Most of the scales used widely and effectively by psychologist are ordinal scales.

Method applied to rank ordered data.

Interval data.

An interval scale is used to specify the magnitude of observations or items.

It is a higher scale of measurements, superior to both nominal and ordinal scales. Thus, it incorporates all the properties of both nominal and ordinal scales and in addition requires that the distance between the classes be equal.

We can apply almost all the usual statistical operations here unless they are of a type that implies knowledge of a true zero point. Even than, the zero point on an interval scale is a matter of convention because the scale form remains in variant when a constant is added.

We can carry out arithmetical operations like addition and subtraction with data on the interval scale.

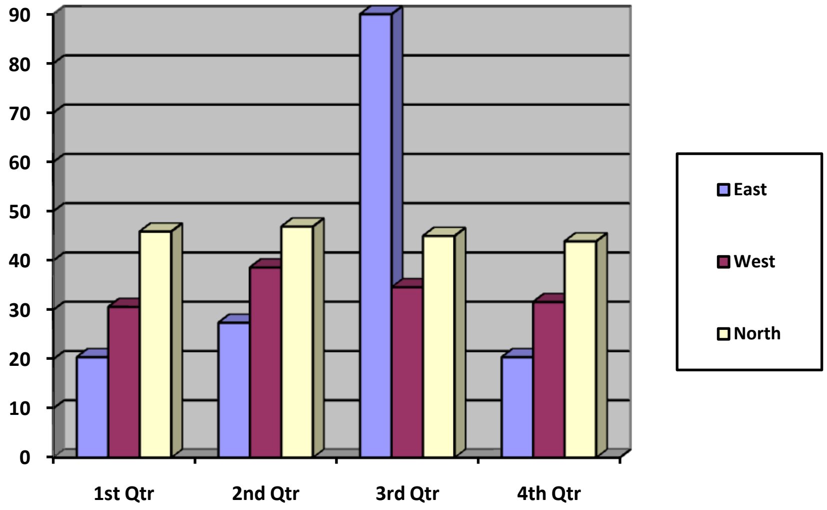
Ratio data.

This is the highest scale of measurement that we shall come across in the physical and natural sciences. Its conditions are equality of rank order, equality of intervals and equality of ratios. The knowledge of the zero point is also a necessary requirement of measurement. All mathematical operations are applicable to the ratio scale and all types of statistical measures are also applicable.

Exercise/Practical

1. Discuss and compare the various scales of measurement.
2. What is the difference between a qualitative and a quantitative variable.

Week Two



2.0 Bar Chart

In bar chart, there are no set of rules to be observed in drawing bar charts. The following consideration will be quite useful.

Note: Bar chart is applicable only to discrete, Categorical, nominal and ordinal data.

1. Bar should be neither too short and nor very long and narrow.
2. Bar should be separated by spaces which are about one and half of the width of a bar.
3. The length of the bar should be proportional to frequencies of the categories.
4. Guide note should be provided to ease the reading of the chart.

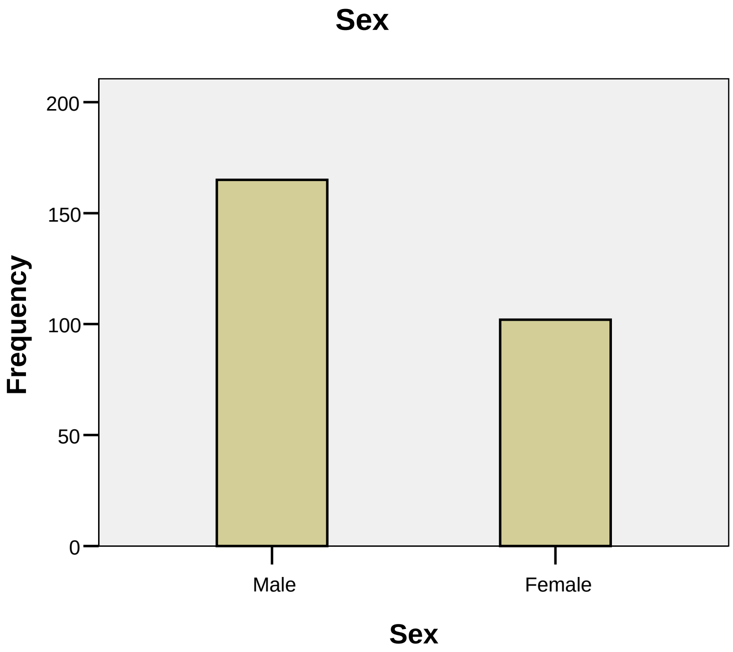
Bar charts are used for making comparisons among categories. In the simplest form several items are presented graphically by horizontal or vertical bars of uniform width, with lengths proportional to the values they represent.

Simple Bar chart

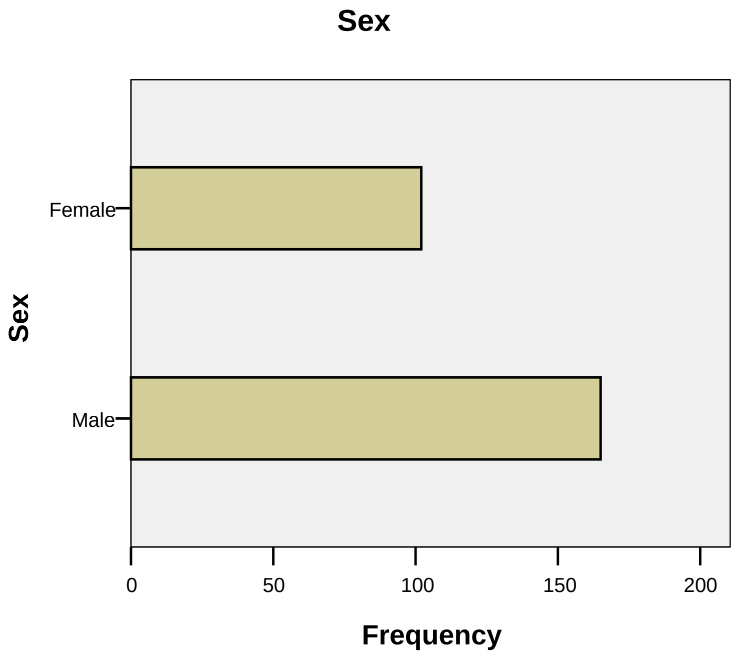
Example

Frequencies

Sex	Frequency
Male	165
Female	102
Total	267



Vertical Approach



Horizontal Approach

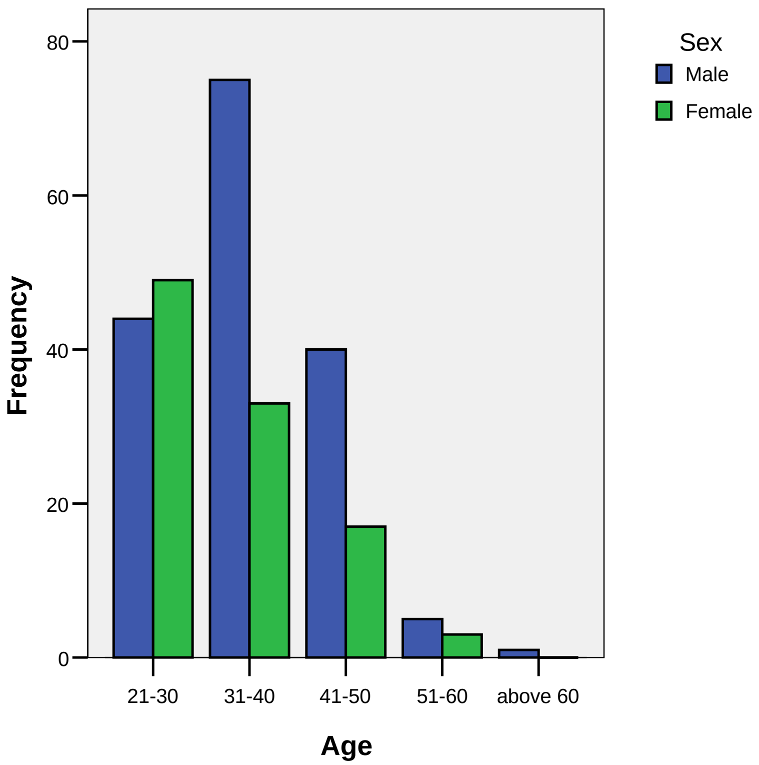
Multiple Bar Charts

These charts enable comparisons of more than one variable to be made the same time. For example, one could go further by considering Age and sex.

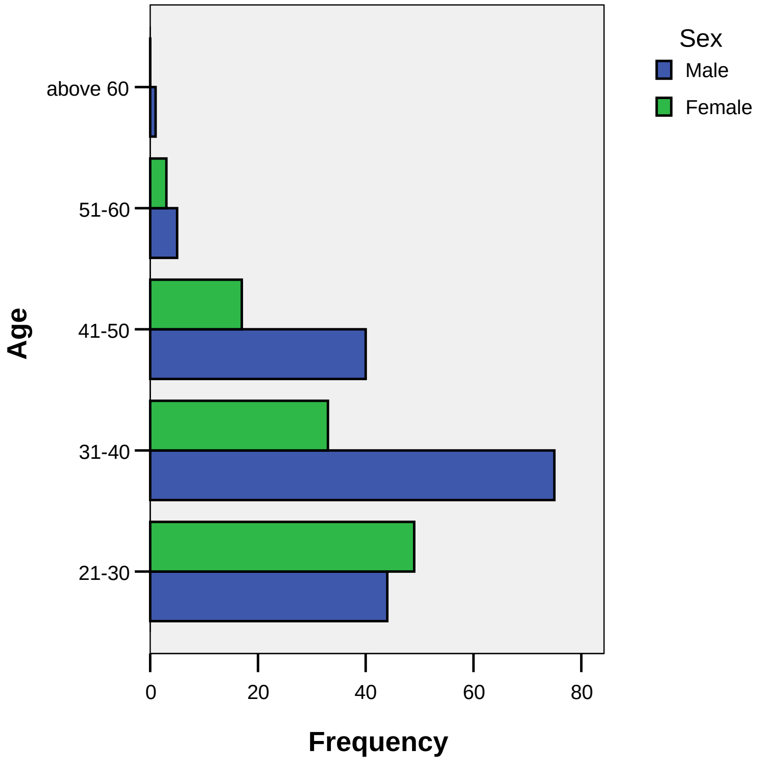
Example

Age(group)	Sex Male	 Female	Total
21-30	44	49	93
31-40	75	33	108
41-50	40	17	57
51-60	5	3	8
above 60	1	0	1
Total	165	102	267

Graph of Multiple Bar Chart



Vertical Approach



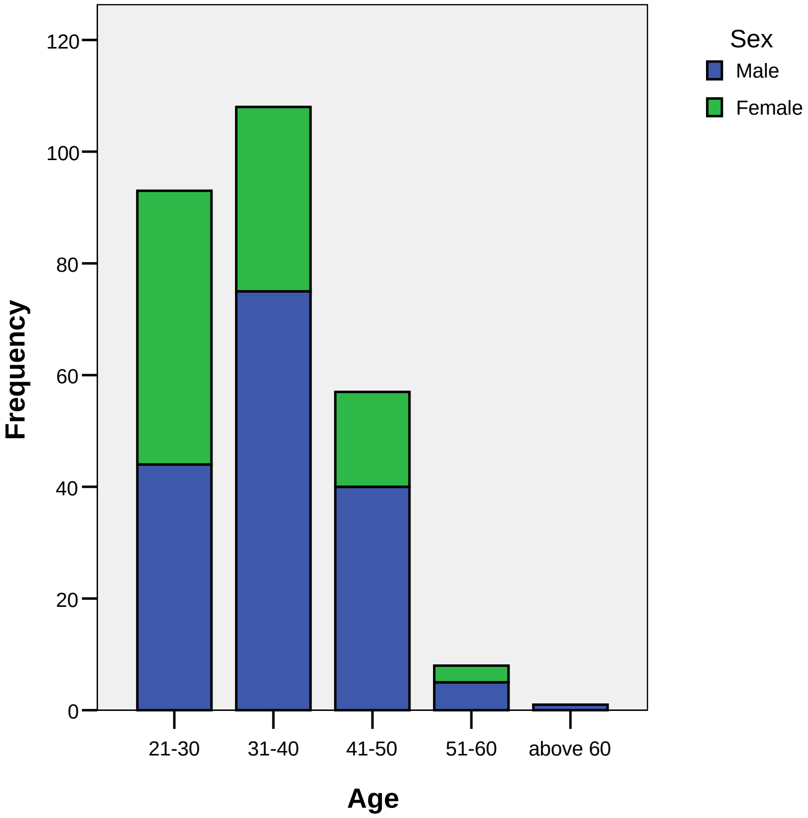
Horizontal Approach

Component Bar Chart

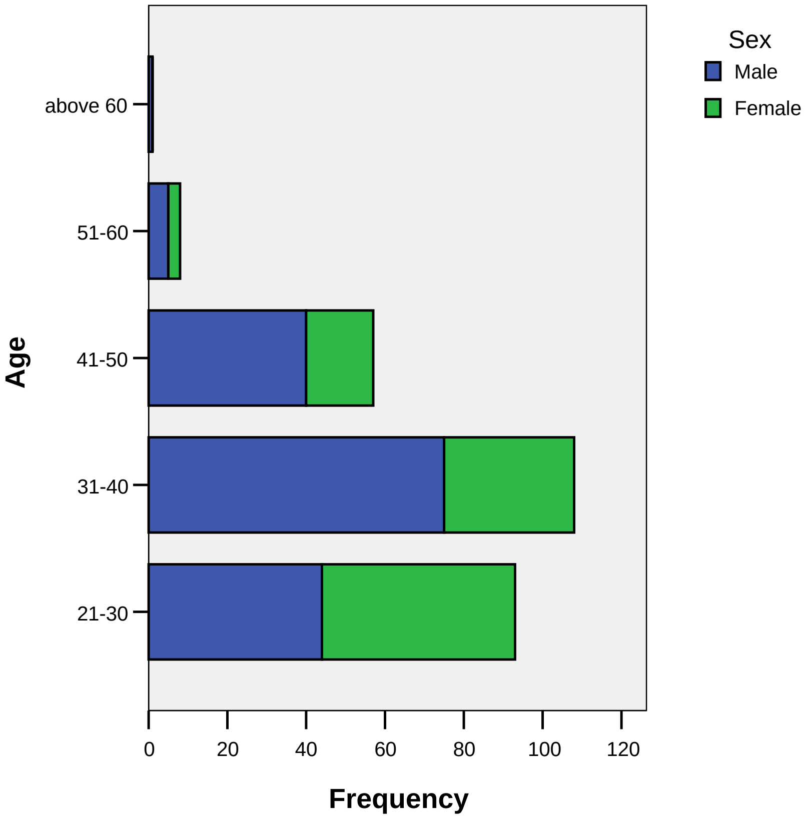
Similarly,these charts enable comparisons of more than one variable to be made the same time. For example, one could go further by considering Age and sex.

Example

Age(group)	Sex		Total
	Male	Female	
21-30	44	49	93
31-40	75	33	108
41-50	40	17	57
51-60	5	3	8
above 60	1		1
Total	165	102	267



Vertical Approach



Horizontal Approach

Exercise/Practical

The data below comes from a survey of physiotherapists in Nigeria and they were asked the questions about patients who have Osteoarthritis knee. And the questions asked were

What age group are you and sex ?

For how long have you been practicing physiotherapy?

In a typical week, how many patients do you see?

On the average, about how many minutes do you spend in treating a patient ?

1. Create simple bar charts for age group and for sex.
2. Create a multiple bar chart for age group with the bars divided into sex
3. Create a component bar chart for age group with sex as the two component
4. For years of practice, suggest why we did not draw a bar chart

S/No	Age group	Sex	Years of practice	Typical	Ave.
1	31-40	Female	4	2	30
2	31-40	Male	14	20	45
3	21-30	Female	8	3	45
4	21-30	Male	3	5	55
5	31-40	Female	10	25	25
6	31-40	Male	10	15	30
7	31-40	Female	9	30	30
8	21-30	Female	2	150	20
9	31-40	Female	2	100	15
10	41-50	Male	17	40	45
11	21-30	Male	5	40	20
12	41-50	Male	17	15	15
13	21-30	Male	3	55	30
14	31-40	Male	11	20	20
15	31-40	Female	10	25	20
16	31-40	Male	3	15	60
17	31-40	Male	14	10	40
18	21-30	Female	2	9	45
19	51-60	Female	29	12	45
20	31-40	Male	6	10	45
21	21-30	Female	4	50	30
22	21-30	Female	5	12	35
23	21-30	Female	.	30	15
24	31-40	Female	18	50	40
25	31-40	Male	5	20	45
26	21-30	Male	5	15	30
27	31-40	Male	10	1	30
28	41-50	Male	13	10	45
29	21-30	Female	2	20	15
30	31-40	Male	7	22	30
31	31-40	Female	13	40	30
32	41-50	Male	22	40	40
33	21-30	Female	8	75	20
34	31-40	Male	9	5	20
35	31-40	Male	7	30	30

36	41-50	Male	20	13	30
37	31-40	Male	5	200	40
38	41-50	Female	24	10	20
39	31-40	Male	3	30	45
40	41-50	Female	16	30	15
41	21-30	Male	3	60	45
42	31-40	Female	11	5	20
43	31-40	Male	7	25	30
44	51-60	Male	25	3	30
45	21-30	Female	4	20	25
46	21-30	Female	3	30	30
47	31-40	Female	11	30	30
48	21-30	Male	3	4	30
49	21-30	Male	5	60	30
50	31-40	Female	16	92	60
51	21-30	Female	7	45	30
52	21-30	Male	3	10	20
53	21-30	Female	4	5	30
54	41-50	Male	16	7	30
55	31-40	Male	10	225	25
56	41-50	Male	17	40	60
57	31-40	Male	15	40	25
58	21-30	Male	2	15	40
59	21-30	Female	1	7	80
60	21-30	Female	2	2	180

Week Three

Pie Chart

A pie chart (or a circle graph) is a circular chart divided into sectors, illustrating relative magnitudes or frequencies. In a pie chart, the arc length of each sector (and consequently its central angle and area), is proportional to the quantity it represents. Together, the sectors create a full disk. It is named for its resemblance to a pie which has been sliced.

While the pie chart is perhaps the most ubiquitous statistical chart in the business world and the mass media, it is rarely used in scientific or technical publications. It is one of the most widely criticized charts, and many statisticians recommend to avoid its use altogether pointing out in particular that it is difficult to compare different sections of a given pie chart, or to compare data across different pie charts. Pie charts can be an effective way of displaying information in some cases, in particular if the intent is to compare the size of a slice with the whole pie, rather than comparing the slices among them. Pie charts work particularly well when the slices represent 25 or 50% of the data, but in general, other plots such as the bar chart or the dot plot, or non-graphical methods such as tables, may be more adapted for representing information.

A pie chart gives an immediate visual idea of the relative sizes of the shares as a whole. It is a good method of representation if one wishes to compare a part of a group with the whole group. You could use a pie chart to show sex of respondents in a given study, market share for different brands or different types of sandwiches sold by a store.

Statisticians tend to regard pie charts as a poor method of displaying information. While pie charts are common in business and journalism, they are uncommon in scientific literature. One reason for this is that it is more difficult for comparisons to be made between the size of items in a chart when area is used instead of length.

However, if the goal is to compare a given category (a slice of the pie) with the total (the whole pie) in a single chart and the multiple is close to 25% or 50%, then a pie chart works better than a graph.

However, pie charts do not give very detailed information, but you can add more information into pie charts by inserting figure into each segment of the chart or by giving a separate table as reference. A pie chart is not a good format for showing increases or decreases numbers in each category, or direct relationships between numbers where our set of numbers depend on another. In this case a line graph would be better format to use.

In order to draw a pie chart you must have data for which you need to show the proportion of each category as a part of the whole. Then the process is as below.

1. Collect the data so the number per category can be counted. In other words, decide on the data that you wish to represent and collect it altogether in a format that shows shares of the whole.
2. Decide on clear title. The title should be a brief description of the data you wish to show. For example, if you wish to show sex of the respondents you could call the pie chart 'sex of the respondent in the study '.
3. Decide on the total number of responses. the number of categories is two (male and female).
4. Calculate the degree share in each category.

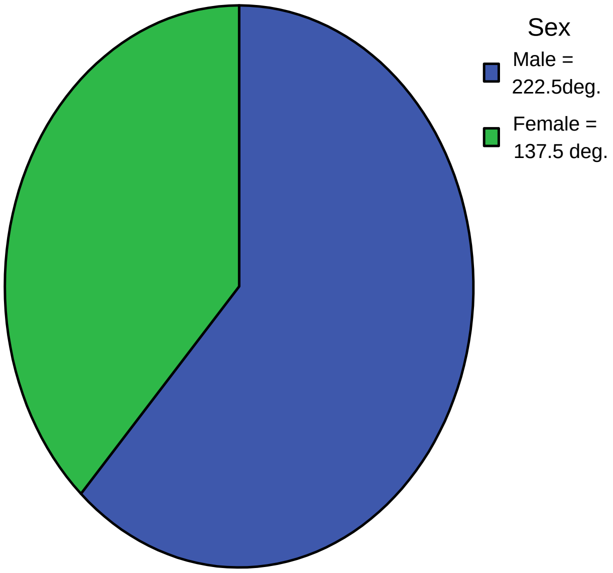
As an example, here is the calculation of the degree share for the sex of the respondents in a given study.

Example 1

Sex of the respondent	Frequency
Male	165
Female	102
Total	267

Angle for male = $\dfrac{\textit{Number of } \text{male}}{\text{Total number}} \times 360 = \dfrac{165}{267} \times 360 = 222.5^0$

Angle for female = $\dfrac{\textit{Number of } \text{female}}{\text{Total number}} \times 360 = \dfrac{102}{267} \times 360 = 137.5^0$



Exercise/Practice

The data below comes from a survey of physiotherapists in Nigeria and they were asked the questions about patients who have Osteoarthritis knee. And the questions asked were

What age group are you and sex ?

For how long have you been practicing physiotherapy?

In a typical week, how many patients do you see?

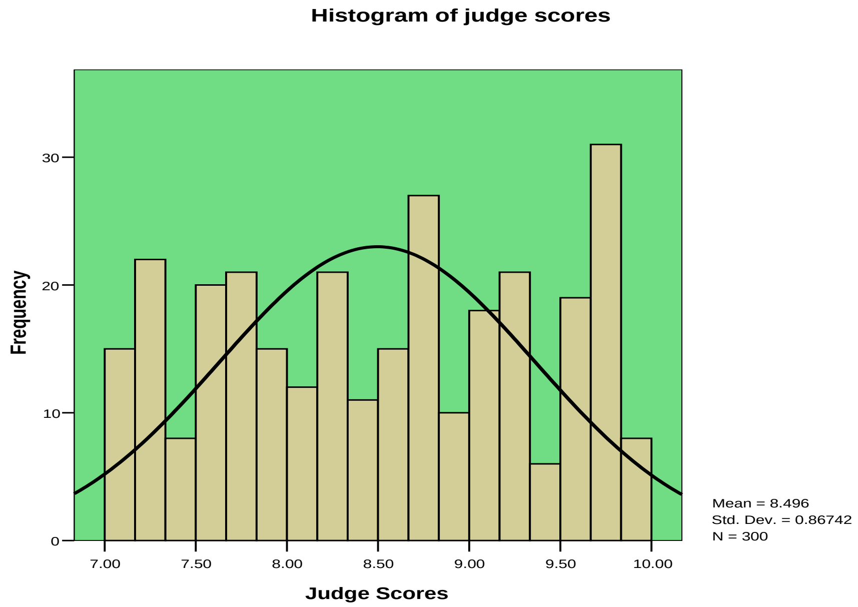
On the average, about how many minutes do you spend in treating a patient ?

a. Create a pie chart for age group of the physiotherapists
b. Create a pie chart for sex of the physiotherapists

S/No	Age group	Sex	Years of practice	Typical	Ave.
1	31-40	Female	4	2	30
2	31-40	Male	14	20	45
3	21-30	Female	8	3	45
4	21-30	Male	3	5	55
5	31-40	Female	10	25	25
6	31-40	Male	10	15	30
7	31-40	Female	9	30	30
8	21-30	Female	2	150	20
9	31-40	Female	2	100	15
10	41-50	Male	17	40	45
11	21-30	Male	5	40	20
12	41-50	Male	17	15	15
13	21-30	Male	3	55	30
14	31-40	Male	11	20	20
15	31-40	Female	10	25	20
16	31-40	Male	3	15	60
17	31-40	Male	14	10	40
18	21-30	Female	2	9	45
19	51-60	Female	29	12	45
20	31-40	Male	6	10	45
21	21-30	Female	4	50	30
22	21-30	Female	5	12	35
23	21-30	Female	10	30	15
24	31-40	Female	18	50	40
25	31-40	Male	5	20	45
26	21-30	Male	5	15	30
27	31-40	Male	10	1	30
28	41-50	Male	13	10	45
29	21-30	Female	2	20	15
30	31-40	Male	7	22	30
31	31-40	Female	13	40	30
32	41-50	Male	22	40	40
33	21-30	Female	8	75	20
34	31-40	Male	9	5	20
35	31-40	Male	7	30	30
36	41-50	Male	20	13	30
37	31-40	Male	5	200	40
38	41-50	Female	24	10	20
39	31-40	Male	3	30	45
40	41-50	Female	16	30	15
41	21-30	Male	3	60	45
42	31-40	Female	11	5	20

43	31-40	Male	7	25	30
44	51-60	Male	25	3	30
45	21-30	Female	4	20	25
46	21-30	Female	3	30	30
47	31-40	Female	11	30	30
48	21-30	Male	3	4	30
49	21-30	Male	5	60	30
50	31-40	Female	16	92	60
51	21-30	Female	7	45	30
52	21-30	Male	3	10	20
53	21-30	Female	4	5	30
54	41-50	Male	16	7	30
55	31-40	Male	10	225	25
56	41-50	Male	17	40	60
57	31-40	Male	15	40	25
58	21-30	Male	2	15	40
59	21-30	Female	1	7	80
60	21-30	Female	2	2	180

Week Four



Histogram

This is the most widely used graphical presentation of a frequency distribution. The histogram is a development of the simple bar chart, with the following differences:

Note: Histogram is applicable only to continuous data. Such as height, weight and so on.

In histogram the bars have to touch each other unlike in bar chart.

1 Except for the case of equal intervals: the area (A) of each rectangular bar is proportional to the frequency in the class, it does not represent its heights. That is A = width * height = frequency.

2 Each rectangular bar is constructed to cover the class it represents without gaps. When constructing the histogram, the following suggestions should be considered:

   (a) Decide on the class intervals

   (b) For each class interval calculate the class frequency

   (c) For unequal interval, find the frequency density in each class by dividing the class frequency by the class interval that is

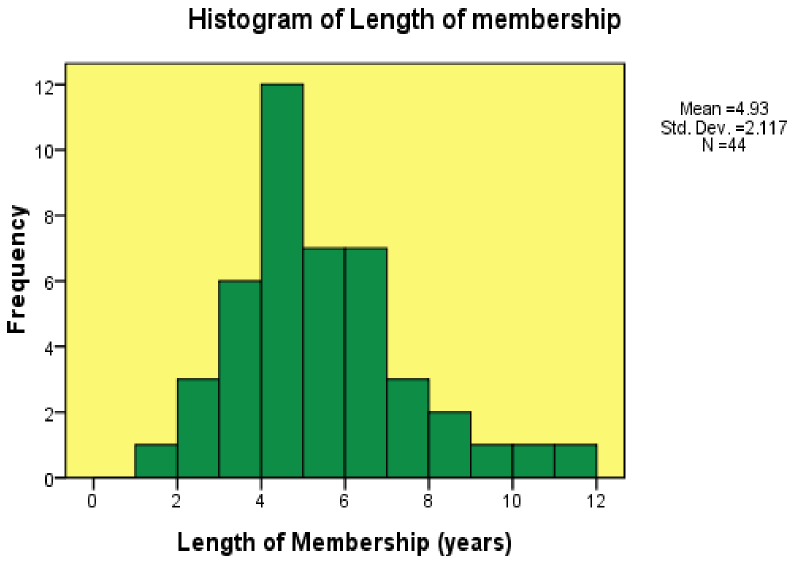
$$d = \frac{\textit{class frequency}}{\textit{class}\ \text{interval}} = \frac{A}{C.I}$$

(d) Use class boundaries and the frequency densities to construct the histogram. For open ended frequency distribution, the class width of the open ended interval should be taken to be equivalent to that of the immediate predecessor.

Note: Histogram is applicable only to continuous data. Such as height, weight and so on.

In histogram the bars have to touch each other unlike in bar chart.

Example



Exercise/Practical

The data below comes from a survey of physiotherapists in Nigeria and they were asked the questions about patients who have Osteoarthritis knee. And the questions asked were

What age group are you and sex ?

For how long have you been practicing physiotherapy?

In a typical week, how many patients do you see?

On the average, about how many minutes do you spend in treating a patient ?

1. Create histogram for years of practice.
2. Create histogram for typical.
3. Create histogram for Average.
4. For sex, suggest why we did not draw histogram.

S/No	Age group	Sex	Years of practice	Typical	Ave.
1	31-40	Female	4	2	30
2	31-40	Male	14	20	45
3	21-30	Female	8	3	45
4	21-30	Male	3	5	55
5	31-40	Female	10	25	25
6	31-40	Male	10	15	30
7	31-40	Female	9	30	30
8	21-30	Female	2	150	20
9	31-40	Female	2	100	15
10	41-50	Male	17	40	45
11	21-30	Male	5	40	20
12	41-50	Male	17	15	15
13	21-30	Male	3	55	30
14	31-40	Male	11	20	20
15	31-40	Female	10	25	20
16	31-40	Male	3	15	60
17	31-40	Male	14	10	40
18	21-30	Female	2	9	45
19	51-60	Female	29	12	45
20	31-40	Male	6	10	45
21	21-30	Female	4	50	30
22	21-30	Female	5	12	35
23	21-30	Female	.	30	15
24	31-40	Female	18	50	40
25	31-40	Male	5	20	45
26	21-30	Male	5	15	30
27	31-40	Male	10	1	30
28	41-50	Male	13	10	45
29	21-30	Female	2	20	15
30	31-40	Male	7	22	30
31	31-40	Female	13	40	30

32	41-50	Male	22	40	40
33	21-30	Female	8	75	20
34	31-40	Male	9	5	20
35	31-40	Male	7	30	30
36	41-50	Male	20	13	30
37	31-40	Male	5	200	40
38	41-50	Female	24	10	20
39	31-40	Male	3	30	45
40	41-50	Female	16	30	15
41	21-30	Male	3	60	45
42	31-40	Female	11	5	20
43	31-40	Male	7	25	30
44	51-60	Male	25	3	30
45	21-30	Female	4	20	25
46	21-30	Female	3	30	30
47	31-40	Female	11	30	30
48	21-30	Male	3	4	30
49	21-30	Male	5	60	30
50	31-40	Female	16	92	60
51	21-30	Female	7	45	30
52	21-30	Male	3	10	20
53	21-30	Female	4	5	30
54	41-50	Male	16	7	30
55	31-40	Male	10	225	25
56	41-50	Male	17	40	60
57	31-40	Male	15	40	25
58	21-30	Male	2	15	40
59	21-30	Female	1	7	80
60	21-30	Female	2	2	180

Week Five

MEASURES OF CENTRAL TENDENCY AND PARTITION

For any set of data, a measure of central tendency is a measure of how the data tends to a central value. It is a typical value such that each individual value in the distribution tends to cluster around it.

In other words, it is an index used to describe the concentration of values near the middle of the distribution. Measures of central tendency are very useful parameters because they describe properties of populations. The word 'average', which is commonly used, refers to the 'centre' of a data set. It is a single value intended to represent the distribution as a whole. Three types of averages are common, they are the mean, the median and the mode.

1.2 THE MEAN

The mean is the most commonly used and also of the greatest importance out of the three averages. There are various types of means. We shall however consider the arithmetic mean, the geometric mean and the harmonic mean.

(A) The arithmetic mean

The arithmetic mean of a series of data is obtained by taking the ratio of the total (sum) of all the data in the series to the number of data points in the series. The arithmetic mean or simply the mean is a representative value of the series that is such that all elements would obtain if the total were shared equally among them.

(a) The mean for ungrouped data

(i) For a set of n items x_1, x_2, x_3, …., x_n, the mean $\bar{x}$ (read x bar)

$$\bar{x} = \frac{\sum x}{n}$$

Where $\sum$ (read: "sigma"), an uppercase Greek letter denotes the summation over values of x and n is the number of values under consideration.

Example

Find the mean of the numbers 3, 4, 6, 7.

Solution

$X_1 = 3$, $X_2 = 4$, $X_3 = 6$, $X_4 = 7$, $N = 4$

$$\bar{x} = \frac{\sum x}{n} = \frac{3+4+6+7}{4} = 20/4 = 5$$

The Coding Method

The coding method sometimes called the assumed mean method is a simplified version of calculating the arithmetic mean. The computational procedure is as follows.

(i) Assume a value within the data set as the mean, that is the assumed mean $(\bar{x}_a)$

(ii) Obtain the deviation of each observation within the data set from the mean.

(iii) Calculate the mean of the deviations from the assumed mean $(\bar{x}_d)$

(iv) Calculate the original mean defined as $\bar{x} = \bar{x}_a + \bar{x}_d$

Example

Calculate the mean of the following numbers 3, 4, 6, 7 using the assumed mean method

Solution

Let the assumed mean $\bar{x}_a = 3$

X	D = x - $\bar{x}_a$
3	0
4	1
6	3
7	4

$$\bar{x}_d = \frac{\sum D}{n} = \frac{0 + 1 + 2 + 3 + 4}{4} = 2.5$$

But $\bar{x} = \bar{x}_a + \bar{x}_d = 3 + 2.5 = 5.5$

(b) The mean for grouped data

If x_1, x_2, x_3, …., x_k, are data points (or midpoints) and f_1, f_2, …, f_k represent the frequencies then,

$$\bar{x} \quad = \quad \frac{f_1 \, x_1 + f_2 \, x_2 + \ldots + f_k \, x_k}{f_1 + f_2 + \ldots + f_k}$$

$$= \quad \frac{\sum fx}{\sum f}$$

Example

The table below shoes the monthly wage of twenty employees of ABC Ventures Ltd.

Monthly wage (₦'000) (x)	No of employees (f)	F x
5	4	20
10	7	70
15	3	45
20	5	100
25	1	25
-	20	260

Solution

$$\bar{x} \quad = \quad \frac{\sum fx}{\sum f} \quad = \quad \frac{260}{20} \quad = \quad 13$$

i.e ₦ 13,000 is the average monthly wage of employees of ABC Ventures Ltd.

Example

The distribution below shows the life – hours of some high powered electric bulbs measured in hundreds of hours

Class Interval	**No of tubes (f)**	**x**	**F x**
1 – 5	5	3	15
6 – 10	15	8	120
11 – 15	18	13	234

16 – 20	20	18	360
21 – 25	25	23	575
26 – 30	9	28	252
31 – 35	5	33	165
36 – 40	3	38	114
Total	**100**	**-**	**1835**

Solution

$$\bar{x} \;=\; \frac{\Sigma fx}{\Sigma f} \;=\; \frac{1835}{100} \;=\; 18.35$$

The short-cut method may be used in computing the arithmetic mean. For a simple frequency distribution,

$$\bar{x} \;=\; \bar{x}_a \;+\; \bar{x}_d \;\text{ where }\; \bar{x}_d = \frac{\Sigma fd}{\Sigma f}$$

For a grouped frequency distribution, with constant factor (i.e equal class interval c) then

$$\bar{x} \;=\; \bar{x}_a \;+\; \bar{x}_d \;\text{ where }\; \bar{x}_d = \left(\frac{\Sigma fd^1}{\Sigma f}\right) \times \text{C}$$

and $d^1 = \dfrac{x - \bar{x}}{C}$

Example

Calculate the mean wage of workers shown in the table below using the assumed mean method

Wage (x)	**No of (f) Employees**	**d= x - $\bar{x}_a$**	**Fd**
5	4	-10	-40
10	7	-5	-35
15	3	0	0
20	5	5	25
25	1	10	10
Total	**20**	-	**-40**

Solution

Take $\bar{x}_a$ = 15

$$\bar{x}_d = \frac{\sum fd}{\sum f} = -\frac{40}{20} = -2$$

But $\bar{x} = \bar{x}_a + \bar{x}_d$

$= 15 - 2 \quad = 13$

Example

Calculate the mean of the distribution below using the assumed mean method.

Class Interval	No of Tubes (f)	Class Mark (x)	$d^1 = \frac{x - \bar{x}_a}{C}$	fd^1
1 – 5	5	3	-4	-20
6 – 10	15	8	-3	-45
11 – 15	18	13	-2	-36
16 – 20	20	18	-1	-20
21 – 25	25	23	0	0
26 – 30	9	28	1	9
31 – 35	5	33	2	10
36 – 40	3	38	3	3
Total	**100**	-	-	**-93**

Take $\bar{x}_a = 23$, $C = 5$

$$\bar{x}_d = C \times \frac{\sum fd}{\sum f} = 5 \times \frac{-93}{100} = -4.65$$

$\therefore \bar{x}_d = 23 - 4.65$

$= 18.35$

Advantages of the arithmetic Mean

(i) It is simple to understand and compute

(ii) It is fully representative since it considers all items observed.

(iii) It can be measured with mathematical exactness. This makes it applicable in advanced statistical analysis.

Disadvantages of the arithmetic Mean

(i) Extreme values affect its result.

(ii) It may not be a physically possible value corresponding to the variable.

(iii) Computational complications may arise for unbounded classes.

(iv) No graphical method can be used to estimate its value.

(v) It is meaningless for qualitative classified data.

Trimmed Mean

The trimmed mean is a family of measures of central tendency. The α % - trimmed mean of N values $x_1, x_2, \ldots, x_N$ is computed by sorting all the N values, discarding α % of the smallest and α % of the largest values, and computing the mean of the remaining values.

For example, to calculate the 20% - trimmed mean for a set of N=5 values (32,10,8,9,11), the following steps are helpful.

Step 1. Sort the values : 8,9,10,11,32.

Step 2. Discard 20% of the largest value – i.e 20% of the largest values –one (20% of 5) largest value (32); discard 20% of the smallest values – i.e one smallest value (8). Then we have a set of three values. (9,10,11)

Step 3. Compute the mean of the three values (9,10,11) is 10.

Thus the 20 % - trimmed mean of 5 values (32,10,8,9,11) is 10.

Arithmetic mean for a set of N=5 values (32,10,8,9,11) is 14

In contrast to the arithmetic mean, the trimmed mean is a robust measure of central tendency. For example, a small fraction of anomalous measurements with abnormally large deviation from the center may change the mean value substantially. At the same time, the trimmed mean is stable in respect to presence of such abnormal extreme values, which get trimmed away.

For example, in the set of 5 values discussed above, replace one value by a large number, say, “12” by “1000” . Then compute the mean of the 5 values, and the 20% - trimmed mean. The replacement does not affect the trimmed mean (because the extreme value is discarded on step 2), but it changes the mean significantly – from 10 to 207.

The trimmed mean, as a family of measures, includes the arithmetic mean and the median as the most extreme case. The trimmed mean with the minimal degree of trimming (α = 0%) coincide with the mean; the trimmed mean with the maximal degree of trimming (α = 50%) coincide with the median.

One popular example of trimmed mean is judges scores in gymnastic, where the extreme scores the underlying distribution is systematic, the truncated mean of a sample is unlikely to produce an unbiased estimator for either the mean or median.

Examples

The scoring method used in many sports that are evaluated by a panel of judges is a truncated mean: *discard the lowest and highest scores; calculate the mean value of the remaining scores.*

The interquartile mean is another example when the lowest 25% and the highest 25% are discarded, and the mean of the remaining scores are calculated.

Exercise

1. The distribution below shows the life – hours of some high powered electric bulbs measured in hundreds of hours. Compute mean

Class Interval	No of tubes (f)
1 – 5	5
6 – 10	15
11 – 15	18
16 – 20	25
21 – 25	25
26 – 30	9
31 – 35	15
36 – 40	3
Total	**120**

2. The number of cars crossing a certain bridge in a big city in intervals of five minutes each were recorded as follows: 20, 15, 16, 30, 20, 20, 12, 9, 18, 15. Calculate the arithmetic mean and trimmed mean. Comment on your results.

WEEK SIX

6.1 THE MEDIAN (measure of central tendency cont'd)

The median of ungrouped data:- The median of a set of data in an array is the value that divides the data set into two equal halves. That is, when these observations are arranged in order of magnitude, half of them will be less than or equal to the median, while the other half will be greater than or equal to it.

The computational procedure for obtaining the median of ungrouped data is as follows:

(i) Arrange the data in order of magnitude (either in increasing or decreasing order)

(ii) Label each observation in that order as $x_{1,}\ x_{2\,---}\ x_n$

(iii) If the number of observations, n is odd, then

$$\text{Median} = X_{\frac{n+1}{2}}$$

If the number of observations n is even, then

$$\text{Median} \quad = \quad \frac{1}{2}\left(X_{\frac{n}{2}} + X_{\frac{n+2}{2}} \right)$$

Example

Compute the median for the following set of numbers

(i) 3 , 6, 8, 9, 7, 12, 2

(ii) 4, 8, 2, 9, 6, 10

Solution

(i) Re-arranging the numbers in ascending order, we have 2, 3, 6, 7, 9, 12

Here n = 7, odd

$x_1 = 2$, $x_2 = 3$, $x_3 = ,6$ $x_4 = 7$, $x_5 = 8$, $x_6 = 9$, $x_7 = 12$

$$\text{Median} = X_{\frac{n+1}{2}}$$

$$\text{Median} = X_{\frac{7+1}{2}}$$

$$= \quad X_4$$

$$= \quad 7$$

(ii) Re-arranging the numbers in ascending order, we have 2, 4, 6, 8, 9, 10

Here n = 6, even and $x_1 = 2$, $x_2 = 4$, $x_3 = 6$, $x_4 = 8$, $x_5 = 9$, $x_6 = 10$

$$\text{Median} = \frac{1}{2}\left(X_{\frac{n}{2}} + X_{\frac{n+2}{2}}\right)$$

$$= \frac{1}{2}\left(X_3 + X_4\right)$$

$$= \frac{1}{2}\left(6 + 8\right) = 7$$

(b) The Median of grouped data:- The median of grouped data can be obtained either by the use of formula or graphically.

(i) The Median by formula.

$$\text{Median} = L_m + \left(\frac{\frac{n}{2} - f_c}{f_m}\right)C$$

Where:

L_m = Low boundary of the median class

n = Total frequencies

$\underline{f_c}$ = Sum of all frequencies before L_m

f_m = frequency of median class

c = class width of median class.

(iii) Graphical Estimate of the Median:- The median of a grouped data can be obtained using the cumulative frequency curve (ogive) and finding from it the value ‘x’ at the 50% point. An effective way of obtaining the median using the graphical method involves converting the frequency values to relative frequencies and expressing it in percentage.

Example

The table below shows the age distribution of employees in a certain factory. Calculate the median age of employees in the factory using the formula and the graphical method.

Age (in yrs.)	No of Employees (f)	Class Boundaries	Cum. Freq.	% Cum Rel. Freq.
20 – 24	2	19.5 – 24.5	2	3
25 – 29	5	24.5 – 29.5	7	12
30 - 34	12	29.5 – 34. 5	19	32
35 – 39	17	34.5 – 39.5	36	60
40 – 44	14	39.5 – 44.5	50	83
45 – 49	6	44.5 – 49.5	56	93
50 – 54	3	49.5 – 54.5	59	98
55 – 59	1	54.5 – 59.5	60	100

(i) By formula:-

$$\text{Median} \quad = \quad L_m \ + \left(\frac{\frac{n}{2} - f_c}{f_m} \right) C$$

$L_m = 34.5$, $n = 60$ $f_c = 19$, $f_m = 17$, $C = 52$

$$\text{Median} \quad = \quad 34.5 \ + \left(\frac{\frac{60}{2} \ - 19}{17} \right) \times 5$$

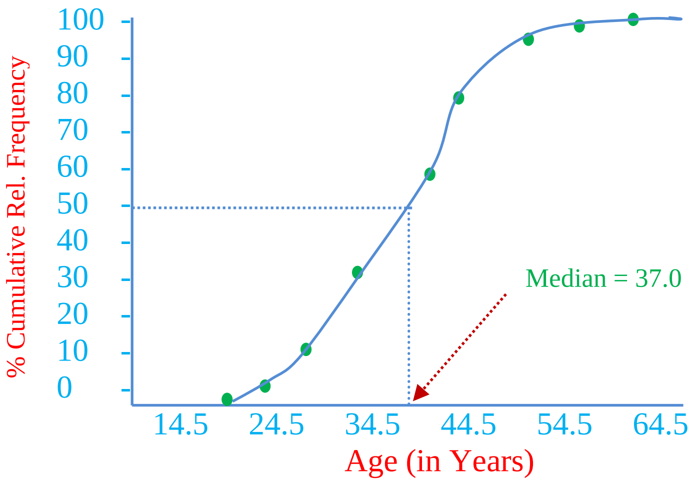
$$= \quad 34.5 + 3.24$$

= 37.74 yrs

(ii) **The graphical approach:-** We note from the last column, that relative % cumulative frequency is

$$\frac{\text{Cum. Frequency}}{\text{Total observations}} \times 100$$

Each of the % cumulative relative frequency is plotted against the corresponding upper class boundary. The median is the value of x at the 50% point shown in the graph below



Advantages of the Median

(i) It is not affected by extreme values

(ii) where there is an odd number of items in an array, the value of the median coincides with one of the items.

(iii) Only the middle items need to be known.

(iv) It is easy to compute.

Disadvantages of the Median

(i) It may not be representative if data items are few

(ii) It is often difficult to arrange in order of magnitude.

(iii) It cannot be used to obtain the total value of items since N * Median $\neq$ total

(iv) In grouped distribution, the median is not an exact value, it is only an estimate.

6.2 MODE

The mode of ungrouped data: For any set of numbers, the mode is that observation which occurs most frequently.

Example

Find the mode of the following numbers.

(i) 2, 5, 3, 2, 6, 2, 2

(ii) 4, 3, 6, 9, 6, 4, 9, 6, 6, 6, 3

Solution

(i) The mode in the first set is 2, it occurs the highest number of times, that is, four times.

(ii) The mode in the second set is 6, with frequency 5

The mode of Grouped Data

The mode of a grouped distribution is the value at the point around which the items tend to be most heavily concentrated. A distribution having one mode, two modes, or more than two modes are called Unimodal, bimodal or multi – modal distribution respectively. In fact, the mode sometimes does not exist if all classes have the same frequency. the mode of grouped data can be obtained either graphically or by use of formula.

(i) The mode by formula

$$\text{Mode} = L_m + \left(\frac{f_m - f_b}{(f_m - f_b) + (f_m - f_a)} \right) C$$

Where

L_m = Lower boundary of modal class

F_m = Frequency of modal class

F_a = Frequency of class immediately after modal class

F_b = Frequency of class immediately before modal class

C = Class width

(ii) Graphical estimate of the mode

The mode of grouped data can be obtained using the histogram

Example

Find the modal age of employees in a factory given in example 3.11 using the formula and the graphical method.

Age (In yrs.)	No. of employees (f)	Class Boundary
20 – 24	2	19.5 – 24.5
25 – 29	5	24.5 – 29.5
30 - 34	12	29.5 – 34. 5
35 – 39	17	34.5 – 39.5
40 – 44	14	39.5 – 44.5
45 – 49	6	44.5 – 49.5
50 – 54	3	49.5 – 54.5
55 – 59	1	54.5 – 59.5

Solution

$$\text{Mode} = L_m + \left(\frac{f_m - f_b}{(f_m - f_b) + (f_m - f_a)}\right) C$$

$L_m = 34.5$ $F_m = 17$, $F_b = 12$, $F_a = 14$, $C = 39.5 – 34.5 = 5$

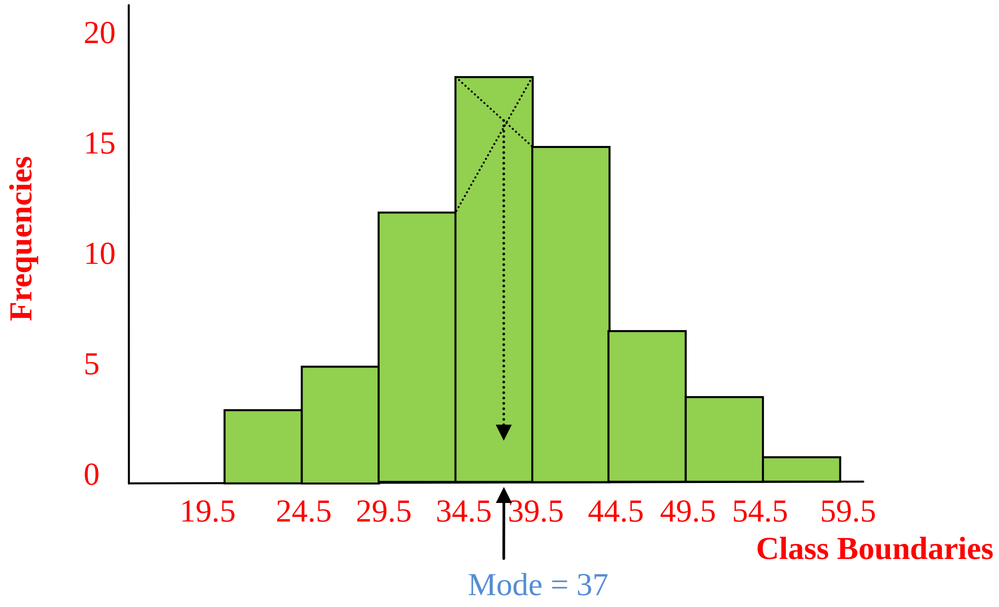
$$\text{Mode} = 34.5 + \left(\frac{17-12}{(17-12)+(17-14)}\right) \times 5$$

$$= 34.5 + \left(\frac{5}{5+3}\right) \times 5$$

$$= 37.63$$

(iii) Graphical Method

Estimation of Mode from Histogram



Advantages of Mode

(i) It is easy to understand and evaluate

(ii) Extreme items do not affect its value

(iii) It is not necessary to have knowledge of all the values in the distribution.

(iv) It coincides with existing items in the observation.

Disadvantages of the Mode

(i) It may not be unique or clearly defined.

(ii) For continuous distribution, it is only an approximation.

(iii) It does not consider all items in the data set.

Exercise /Practical

1. The following data are scores on a management examination taken by a group of 20 people.

   88, 56, 64, 45, 52, 76, 38, 98, 69, 77

   71, 45, 60, 90, 81, 87, 44, 80, 41, 58

   Find the median and mode.

2. Given the data below

   23, 26, 29, 30, 32, 34, 37, 45, 57,80, 102, 147, 210, 355, 782, 1,209

   Find the median and the mode.

3. The following table gives then distribution of marks obtained by 100 students in the college of engineering in a test of engineering drawing.

Marks(%)	10-9	20-29	30-39	40-49	50-59	60-69	70-79
No.of stud.	5	10	14	29	28	10	4

   Use the table to calculate:

   (i) Median (ii) Mode of the grouped data

4. Given the data below

   41 35 27 19 51 47 63 76 22 39

   14 23 18 39 92 61 45 13 37 22

   33 51 53 19 29 72 27 40 57 67

   84 76 91 33 58 73 86 65 43 80

   From a grouped frequency table with the class intervals:

   11-20, 21-30, 31-40….. etc

   Hence use the table to calculate:

   (i) Median (ii) Mode

WEEK Seven

7.0 QUANTILES

All quantities that are defined as partitioning or splitting a distribution into a number of equal portions are called quantiles. Examples include the quartiles, deciles and the percentiles.

The three quantities that spilt a distribution into four equal parts are called **Quartiles,** namely **(Q1),** second quartiles (Q_2) and the third quartiles, (Q_3). Nine quantities spilt a distribution into ten equal parts. These are called Deciles namely first decile (D_1), Second decile (D_2), up to the ninth decile (D_9). The Ninety-nine quantities that spilt a distribution into one hundred equal parts are called percentiles namely first Percentiles (P_1), second Percentile (P_2) up to the ninety-ninth percentile (P_{99}).

7.1 QUARTILES

The quartiles can be obtained either by formula or by using the cumulative frequency curve. The calculation of the quartiles for both ungrouped and grouped data is similar to parallel calculations of the median for ungrouped and grouped data using appropriately modified versions. The formula for obtaining some quartiles are shown below

$$Q_1 = \mathrm{L}_1 + \left(\frac{\frac{n}{4} - f_c}{f_1} \right) C$$

$$Q_2 = \mathrm{L}_2 + \left(\frac{\frac{2n}{4} - f_c}{f_2} \right) C$$

$$Q_3 = \mathrm{L}_3 + \left(\frac{\frac{3n}{4} - f_c}{f_3} \right) C$$

7.2 DECILES

The formula for obtaining some Deciles are shown below:

$$D_1 \quad = \mathrm{L}_1 + \left(\frac{\frac{n}{10} - f_c}{f_1} \right) C$$

$$D_2 \quad = \mathrm{L}_2 + \left(\frac{\frac{2n}{10} - f_c}{f_2} \right) C$$

“ “ “ “ “
“ “ “ “ “

$$D_9 \quad = \mathrm{L}_9 + \left(\frac{\frac{9n}{10} - f_c}{f_9} \right) C$$

7.3 PERCENTILES

The formula for obtaining some Percentiles are shown below:

$$P_1 \quad = \mathrm{L}_1 + \left(\frac{\frac{n}{100} - f_c}{f_1} \right) C$$

$$P_2 \quad = \mathrm{L}_2 + \left(\frac{\frac{2n}{100} - f_c}{f_2} \right) C$$

“ “ “ “ “
“ “ “ “ “

$$P_{99} \quad = \quad L_{99} \; + \; \left(\frac{\frac{99n}{100} - f_c}{f_{99}} \right) C$$

Note : All the equations above have the same definition as used in the median.

Example

Consider the age distribution of employees in a factory given in example 3.11 calculate

(a) The first and third quartile

(b) The second, fourth and ninth deciles

(c) The tenth, fiftieth and ninetieth percentiles

Use both the formula and the graphical method

Solution

By formula,

A. $$Q_1 \quad = \quad L_1 \; + \; \left(\frac{\frac{n}{4} - f_c}{f_1} \right) C$$

$$= \; 29.5 \; + \; \left(\frac{15 - 7}{12} \right) \times 5$$

$$= \; 32.8 \text{ yrs}$$

$$Q_3 \quad = \quad L_3 \; + \; \left(\frac{\frac{3n}{4} - f_c}{f_3} \right) C$$

$$= \; 39.5 \; + \; \left(\frac{45 - 36}{14} \right) \times 5$$

$$= \; 42.7 \text{ yrs}$$

B. $$D_2 = \mathrm{L}_2 + \left(\frac{\frac{2n}{10} - f_c}{f_2} \right) C$$

$$= 29.5 + \left(\frac{12-7}{12} \right) \times 5$$

$$= 31.6 \text{ yrs}$$

$$D_4 = \mathrm{L}_4 + \left(\frac{\frac{4n}{10} - f_c}{f_4} \right) C$$

$$= 34.5 + \left(\frac{24-19}{17} \right) \times 5$$

$$= 36.0 \text{ yrs}$$

$$D_9 = \mathrm{L}_9 + \left(\frac{\frac{9n}{10} - f_c}{f_9} \right) C$$

$$= 44.5 + \left(\frac{54-50}{6} \right) \times 5$$

$$= 47.8 \text{ yrs}$$

C. $$P_{10} = \mathrm{L}_{10} + \left(\frac{\frac{10n}{100} - f_c}{f_{10}} \right) C$$

$$= 24.5 + \left(\frac{6-2}{5} \right) \times 5$$

$$= 28.5 \text{ yrs}$$

$$P_{50} \quad = \quad \mathrm{L}_{50} + \left(\frac{\frac{50n}{100} - f_c}{f_{50}} \right) C$$

$$= \; 34.5 + \left(\frac{30 - 19}{17} \right) \times 5$$

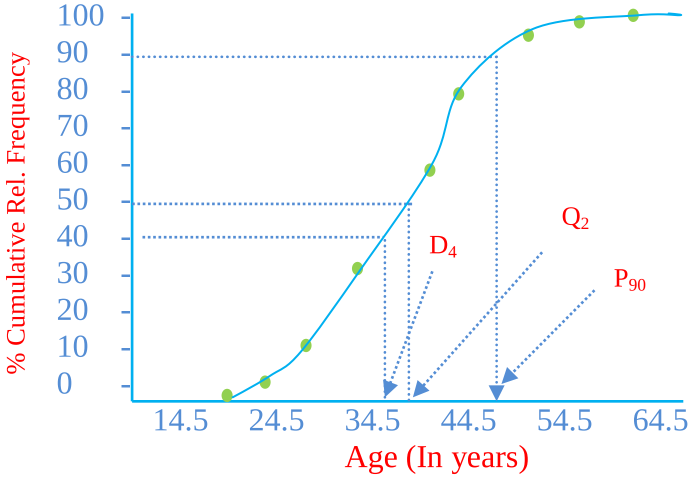
$$= \; 37.7 \text{ yrs}$$

$$P_{90} \quad = \quad \mathrm{L}_{90} + \left(\frac{\frac{90n}{100} - f_c}{f_{90}} \right) C$$

$$= \; 44.5 + \left(\frac{54 - 50}{6} \right) \times 5$$

$$= \; 47.8 \text{ yrs}$$

Cumulative Frequency Curve (Ogive) showing estimation of quartiles



From the ogive, the required points are located as follows

$Q_1 \longrightarrow \frac{1}{4} \times 100 = 25$

$Q_2 \longrightarrow \frac{3}{4} \times 100 = 75$

$D_2 \longrightarrow \frac{2}{10} \times 100 = 20$

$P_{10} \longrightarrow \frac{10}{100} \times 100 = 10$ e.t.c

WEEK Eight

8.0 MEASURES OF DISPERSION

A measure of dispersion is a measure of the tendency of individual values of the variable to differ in size among themselves. In summarizing a set of data, it is generally desirable not only to indicate its average but also to specify the extent of clustering of the observations around the average. Measures of variability provide an indication of how well or poorly measures of central tendency represent a particular distribution. If a measure of dispersion is for instance, zero, there is no variability among the values and the mean is perfectly representative. In general, the greater the variability, the less representative the measure of central tendency.

Some important measures of dispersion include the range, semi- inter-quartile range, mean deviation, variance and standard deviation.

8.1 RANGE

The range R, of a set of numbers is the difference between the largest and smallest numbers, that is, it is the difference between the two extreme values. Suppose $X_L - X_S$

In a grouped frequency distribution the midpoint of the first and last class are chosen as X_L and X_S respectively.

Example

Compute the range for the following numbers; 6, 9, 5, 18, 25

Solution:

$$R = X_L - X_S$$

Where $X_L = 25, \quad X_S = 5$

$$\therefore \text{range} = 25 - 5$$
$$= 20$$

8.2 QUARTILE DEVIATION

For any set of data, the quartile deviation or semi-interquartile range is defined as half the difference between the third and first quartile, that is,

Q.D = ½ $(Q_3 - Q_1)$

The third quartile (Q_3) and the first quartile (Q_1) are obtained as discussed in chapter three.

8.3 MEAN DEVIATION

For any set of numbers $x_1, x_2, \ldots, x_n$, the mean deviation (M.D) is defined as follows.

$$\text{M.D} = \frac{\sum |x - \bar{x}|}{n}$$

$$\text{where } \bar{x} = \frac{\sum x}{n}$$

$$\frac{|x - \bar{x}|}{n} = \text{absolute value of the difference between } x_1 \text{ and } \bar{x}$$

If $x_1, x_2 \ldots\ldots, x_k$ is repeated with frequency $f_1, f_2 \ldots\ldots\ldots f_k$ then

$$\text{M.D} = \frac{\sum f\ |x - \bar{x}|}{\sum f}$$

$$\text{Where } \bar{x} = \frac{\sum fx}{\sum f}$$

Example

Calculate the mean deviation for the following set of numbers.

(i) 3, 5, 6, 7, 4

(ii) 10, 25, 35, 40, 20, 30, 45, 55, 15, 25

Solution:

(i) $\bar{x} = \dfrac{\sum x}{n}$

$$= \frac{3+5+6+7+4}{5} = \frac{25}{5} = 5$$

X	**x** $- \bar{x}$	$\lvert x - \bar{x} \rvert$
3	-2	2
5	0	0
6	1	1
7	2	2
4	-1	1
Total	**-**	**6**

$$\text{M.D} = \frac{\sum \lvert x - \bar{x} \rvert}{n}$$

$$= \frac{6}{5} = 1.2$$

(ii) $$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{10 + 25 + \ldots + 25}{10}$$

$$= \frac{300}{10}$$

$$= 30$$

X	**x** $- \bar{x}$	$\lvert x - \bar{x} \rvert$
10	-20	20
25	-5	5
35	5	5
40	10	10
20	-10	10
30	0	0
45	15	15
55	25	25
15	-15	15
25	5	5
Total	**-**	**110**

$$\text{M.D} \quad = \quad \frac{\sum |x - \bar{x}|}{n}$$

$$= \quad \frac{110}{10}$$

$$= \quad 11$$

Example
Calculate the mean deviation for the distribution below

Class interval	Freq. (*f*)	X	*fx*	$x - \bar{x}$	$\lvert x - \bar{x}\rvert$	$f\lvert x - \bar{x}\rvert$
1 -3 4 -6 7 – 9 10 – 12 13 – 15	5 10 15 10 5	2 5 8 11 14	10 50 120 110 70	-6 -3 0 3 6	6 3 0 3 6	30 30 0 30 30
Total	**45**		**360**			**120**

$$\bar{x} \quad = \quad \frac{\sum fx}{\sum f} \quad = \quad \frac{360}{45} \quad = \quad 8$$

$$\text{M.D} \quad = \quad \frac{\sum f\,|x - \bar{x}|}{\sum f} \quad = \quad \frac{120}{45} \quad = \quad 2.67$$

8.3 VARIANCE AND STANDARD DEVIATION

Instead of merely neglecting the signs of the deviations from the arithmetic mean, we may square the deviations, thereby making them all positive. The measure of dispersion obtained by taking the arithmetic mean of the sum of squared deviations of the individual observations from the mean is called the variance or mean square deviation or simply mean square.

The variance of a set of numbers $x_1, x_2\ldots.., x_n$ denoted by σ^2 is defined as follows:

$$\sigma^2 \quad = \quad \frac{\sum(x-\bar{x})^2}{n-1}, \qquad \text{where} \quad \bar{x} = \frac{\sum x}{n}$$

If $x_1, x_2\ldots.., x_k$ is repeated with frequencies $f_1, f_2, \ldots f_k$ then

$$\sigma^2 = \frac{\sum f(x-\bar{x})^2}{N-1}$$

And standard deviation (SD) $= \sqrt{\frac{\sum \mathrm{f}\ (\mathrm{x} - \bar{\mathrm{x}})^2}{N-1}}$

Where $\bar{x} = \frac{\sum fx}{\sum f}$

Exercise /Practical

1. The following table gives then distribution of marks obtained by 100 students in the college of engineering in a test of engineering drawing.

Marks(%)	10-9	20-29	30-39	40-49	50-59	60-69	70-79
No.of stud.	5	10	14	29	28	10	4

Use the table to calculate:

(i) Standard deviation

2. Given the data below

41	35	27	19	51	47	63	76	22	39
14	23	18	39	92	61	45	13	37	22
33	51	53	19	29	72	27	40	57	67
84	76	91	33	58	73	86	65	43	80

From a grouped frequency table with the class intervals:

11-20, 21-30, 31-40….. etc

Hence use the table to calculate:

(i) Variance

Calculate the mean deviation for the following set of numbers.

3 3, 8, 6, 7, 4,9,10,12,22,11,14.

4 10, 25, 35, 40, 20, 40, 55, 55, 35, 25,20,35,65,75

WEEK Nine

9.0 BOX PLOT

In descriptive statistics, a box plot or box plot (also known as a box-and-whisker diagram or plot) is a convenient way of graphically depicting groups of numerical data through their five-number summaries (the smallest observation (sample minimum), lower quartile (Q1), median (Q2), upper quartile (Q3), and largest observation (sample maximum). A box plot may also indicate which observations, if any, might be considered outliers.

Box plots can be useful to display difference populations without making any assumptions of the underlying statistical distribution: they are non-parametric. The spacing between the different parts of the box help indicate the degree of dispersion (spread) and skewness in the data , and identifying outliers. Box plots can be drawn either horizontally or vertically.

Construction

There are a number of conventions used in drawing box plots; the following is a common one.

For a data set, one constructs a horizontal box plot in the following manner:

Calculate the first LQ ($x_{.25}$), the median ($x_{.50}$) and third quartile ($x_{.75}$)

Calculate the interquartile range (IQR) BY subtracting the first quartile from the third quartile ($x_{.75}$)- ($x_{.25}$).

Construct a box above the number line bounded on the left by the first quartile ($x_{.25}$) and on the right by the third quartile ($x_{.75}$).

Indicate where the median lies inside of the box with the presence of a symbol or a line dividing the box at the median value.

The mean value of the data can also be labeled with a point.

Any data observation which lies more than 1.5 Interquartile range (IQR) lower than the first quartile or 1.5 IQR higher than the third quartile is considered an outlier. Indicate where the smallest value that is not an outlier is by connecting it to the box with a horizontal line or "whisker". Optionally, also mark the position of this value more clearly using a small vertical line. Likewise, connect the largest value that is not an outlier to the box by a "whisker" (and optionally mark it with another small vertical line).

Indicated outliers by open and closed dots. "Extreme" outlier, or those which lie more than three times the IQR (3.IQR) to the left and right from the first and third quartiles respectively, are indicated by the presence of closed dot. “Mild” outlier – that is , those observations which lie more than 1.5 times the IQR from the first and the third quartile but are not also extreme outliers are indicated by the presence of a open dot. (Sometimes no distinction is made between “mild” and “extreme” outliers.)

Add an appropriate label to the number line and title the box plot.

A box plot may be constructed in a similar manner vertically as opposed to horizontally by merely interchanging “bottom” for “top” for “right” and “vertical” for “horizontal” in the above description.

1. smallest non-outlier observation = 5 (left “whisker”) (left “whisker” would have been 4 had there been an observation with a value of 4 (Q1 –-1.5.IQR))
2. lower quartile (Q1, $x_{.25}$) = 7
3. median (Med, $x_{.5}$) = 8.5
4. upper quartile (Q3, $x_{.75}$) = 9
5. largest non-outlier observation = 10 (right “whisker”)
6. interquartiler range, IQR = Q3 – Q1 = 2
7. the value 3.5 is a “mild” outlier, between 1.5.IQR and 3.IQR below Q1
8. the value 0.5 is an “extreme” outlier, more than 3.IQR below Q1
9. the data is skewed to the left (negative skewed)

The horizontal lines (the “whiskers”) extend to at most 1.5 time the box width (the interquartile range) from either or both ends of the box. They must end at an observed value, thus connecting all the values outside the box that are not more than 1.5 times the box width away from box. Three times the box width marks the boundary between “mild” and “extreme” outlier. In this box plot, “mild” and “extreme” outliers are differentiated by closed and open dot, respectively.

There are alternative implementations of this detail of the box plot in various software packages, such as the whiskers extending to at most the 5th and 95th (or some more extreme) percentiles. Such approaches do not conform to Turkeys definition, with its emphasis on the median in particular and counting methods in general, and they tend to produce “outlier” for all data sets larger than ten, no matter what the shape of the distribution.

9.1 ALTERNATIVE FORMS

Box and whisker plots are uniform in their use of the box: the bottom and top of the box are always the 25th and 75th percentile (the lower and upper quartiles, respectively), and the band near the middle of the box is always the 50th percentile (the median). But the ends of the whiskers can represent several possible alternative values, among them:

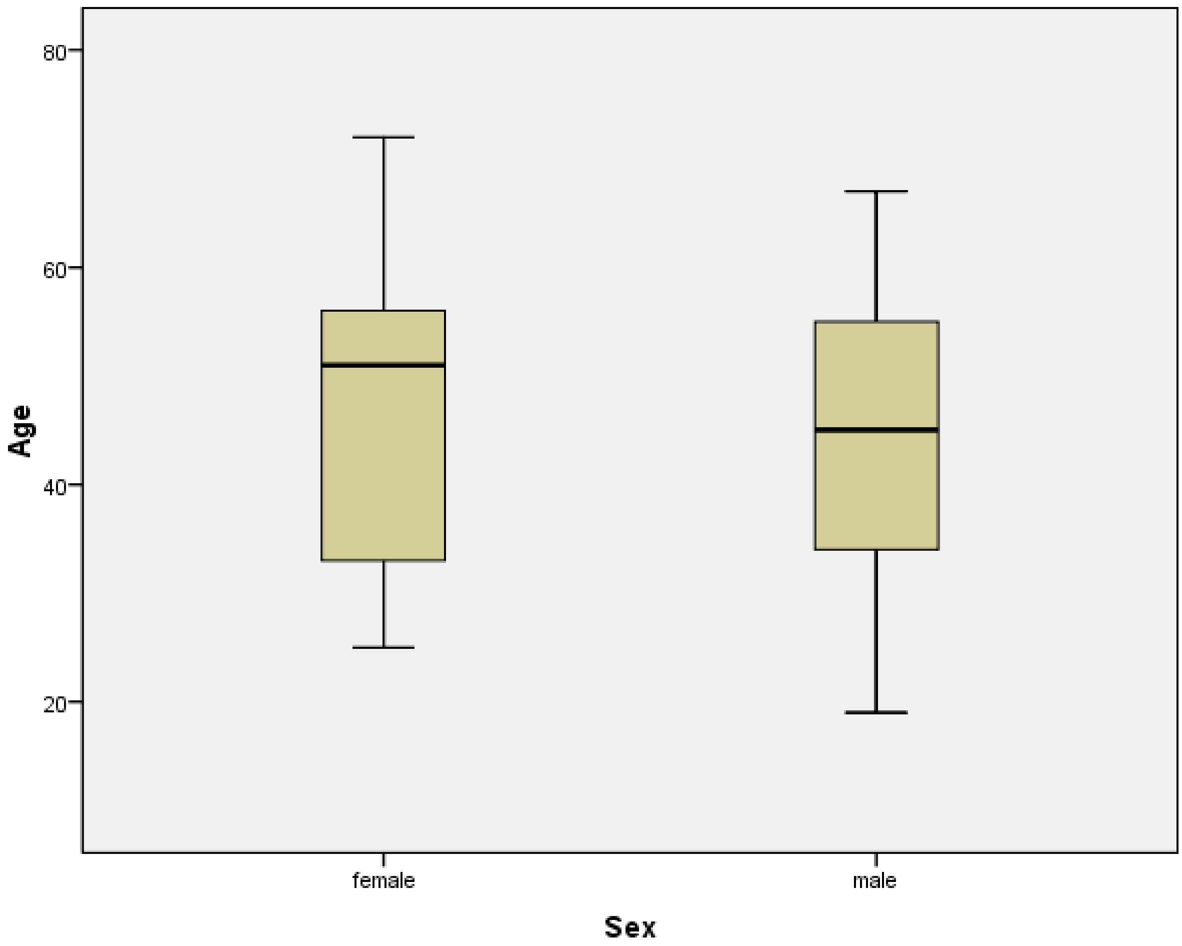
1. the minimum and maximum of all the data
2. the lowest datum still within 1.5 IQR of the lower quartile, and the highest datum still within 1.5 IQR of the upper quartile.
3. One standard deviation above and below the mean of the data
4. the 9th percentile and the 91st percentile
5. the 2nd percentile and the 98th percentile

Any data not included between the whiskers should be plotted as an outlier with a dot, small circle, or star, but occasionally this is not done.

Some box plots include an additional dot or a cross is plotted inside of the box, to represent the mean of the data in the median.

On some box plots a crosshatch is placed on each whisker, before the end of the whisker.

Fairly rarely, box plots can be presented with no whisker



Exercise/Practical

The data below comes from a survey of physiotherapists in Nigeria and they were asked the questions about patients who have Osteoarthritis knee. And the questions asked were

What age group are you and sex ?

For how long have you been practicing physiotherapy?

In a typical week, how many patients do you see?

On the average, about how many minutes do you spend in treating a patient ?

1. Create Box plot for Years of practice and sex.
2. Create Box plot for average and sex.

S/No	Age group	Sex	Years of practice	Typical	Ave.
1	31-40	Female	4	2	30
2	31-40	Male	14	20	45
3	21-30	Female	8	3	45
4	21-30	Male	3	5	55
5	31-40	Female	10	25	25
6	31-40	Male	10	15	30
7	31-40	Female	9	30	30
8	21-30	Female	2	150	20
9	31-40	Female	2	100	15
10	41-50	Male	17	40	45
11	21-30	Male	5	40	20
12	41-50	Male	17	15	15
13	21-30	Male	3	55	30
14	31-40	Male	11	20	20
15	31-40	Female	10	25	20
16	31-40	Male	3	15	60
17	31-40	Male	14	10	40
18	21-30	Female	2	9	45
19	51-60	Female	29	12	45
20	31-40	Male	6	10	45
21	21-30	Female	4	50	30
22	21-30	Female	5	12	35
23	21-30	Female	.	30	15
24	31-40	Female	18	50	40
25	31-40	Male	5	20	45
26	21-30	Male	5	15	30
27	31-40	Male	10	1	30
28	41-50	Male	13	10	45
29	21-30	Female	2	20	15
30	31-40	Male	7	22	30
31	31-40	Female	13	40	30
32	41-50	Male	22	40	40
33	21-30	Female	8	75	20
34	31-40	Male	9	5	20
35	31-40	Male	7	30	30
36	41-50	Male	20	13	30
37	31-40	Male	5	200	40
38	41-50	Female	24	10	20
39	31-40	Male	3	30	45

40	41-50	Female	16	30	15
41	21-30	Male	3	60	45
42	31-40	Female	11	5	20
43	31-40	Male	7	25	30
44	51-60	Male	25	3	30
45	21-30	Female	4	20	25
46	21-30	Female	3	30	30
47	31-40	Female	11	30	30
48	21-30	Male	3	4	30
49	21-30	Male	5	60	30
50	31-40	Female	16	92	60
51	21-30	Female	7	45	30
52	21-30	Male	3	10	20
53	21-30	Female	4	5	30
54	41-50	Male	16	7	30
55	31-40	Male	10	225	25
56	41-50	Male	17	40	60
57	31-40	Male	15	40	25
58	21-30	Male	2	15	40
59	21-30	Female	1	7	80
60	21-30	Female	2	2	180

WEEK Ten

10.0 COMPUTATION OF VARIANCE AND STANDARD DEVIATION

The sample variance of a set of numbers x_1, x_2….., x_n denoted by σ^2 is defined as follows:

$$\sigma^2 = \frac{\sum(x-\bar{x})^2}{n-1}, \qquad \text{where} \quad \bar{x} = \frac{\sum x}{n}$$

If x_1, x_2….., x_k is repeated with frequencies $f_1, f_2, \dots f_k$ then

$$\sigma^2 = \frac{\sum f(x-\bar{x})^2}{N-1}$$

And standard deviation is $\sqrt{\sigma^2}$

Where $\bar{x} = \frac{\sum fx}{\sum f}$

Example

Calculate the variance and standard deviation for the following set of numbers

(i) 3,5, 6, 7, 4

(ii) 10, 25, 35, 40, 20, 30, 45, 55, 15, 25

Solution;

(i) $\bar{x} = \frac{\sum x}{n}$

$= \frac{25}{5}$

$= 5$

X	**$(x - \bar{x})$**	**$(x - \bar{x})^2$**
3	-2	4
5	0	0
6	1	1
7	2	4
4	-1	1
Total	**-**	**10**

$$\sigma^2 = \frac{\sum(x-\bar{x})^2}{n-1}$$

$$= \frac{10}{4} = 2.5$$

(ii) $$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{300}{10} = 30$$

X	**(x – $\bar{x}$)**	**(x – $\bar{x}$)^2**
10	-20	400
25	-5	25
35	5	25
40	10	100
20	-10	100
30	0	0
45	15	225
55	25	625
15	-15	225
25	-5	25
Total	**-**	**1750**

$$\sigma^2 = \frac{\sum(x-\bar{x})^2}{n-1}$$

$$= \frac{1750}{9}$$

$$= 194.44$$

Example

Calculate the variance for the distribution below

Class interval	Freq. (*f*)	x	fx	$x - \bar{x}$	$(x - \bar{x})$	$f(x - \bar{x})^2$
1 -3	5	2	10	-6	36	180
4 -6	10	5	50	-3	9	90
7 – 9	15	8	120	0	0	0
10 – 12	10	11	110	3	9	90
13 – 15	5	14	70	6	36	180
Total	**45**	**-**	**360**	**-**	**-**	**540**

$$\bar{x} = \frac{\sum x}{\sum f}$$

$$= \frac{360}{45} = 8$$

$$\sigma^2 = \frac{\sum f(x-\bar{x})^2}{N-1} = \frac{540}{45} = 12.27$$

Computational formula could be obtained from the definition of the variance. these are

$$\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 .$$ for ungrouped data and,

$$\sigma^2 = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2 .$$ for grouped data

Example

Use the computational formula to calculate the variance for the following set of numbers.

(i) 3, 5, 6, 7, 4

(ii) 10, 25, 35, 40, 20, 30, 45, 55, 15, 25

Solution:

(i) $\sum$ x = 3 + 5+ 6+ 7 + 4 = 25

$\sum x^2 = 3^2 + 5^2 + 6^2 + 7^2 + 4^2 = 135$

$$\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 .$$

$$= \frac{135}{5} - \left(\frac{25}{5}\right)^2 .$$

$= 27 - 25$
$= 2$

(ii) $\sum x = 10 + 25 + \ldots + 25 = 300$

$\sum x^2 = 10^2 + 25^2 + \ldots + 25^2 = 10750$

$$\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 .$$

$= 1075 - 900 = 175$

Example:

Use the computational formula to calculate the variance for the distribution below:

Class Interval	Frequency *(f)*	X	x^2	*F*x	fx^2
1-3	5	2	4	10	20
4 – 6	10	5	25	50	250
7 – 9	15	8	64	120	960
10 – 12	10	11	121	110	1210
13 – 15	5	14	196	70	980
Total	**45**	**-**	**-**	**360**	**3420**

$\sum f = 45$

$\sum f\text{x} = 360$

$\sum f\text{x}^2 = 3420$

$$\sigma^2 = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2 .$$

$$= \frac{3420}{45} - \left(\frac{360}{45}\right)^2 .$$

= 76 - 64

= 12

Exercise/Practical

1. The following table gives then distribution of marks obtained by 100 students in the college of engineering in a test of engineering drawing.

Marks(%)	10-9	20-29	30-39	40-49	50-59	60-69	70-79
No. of students.	5	10	14	29	28	10	4

Use the table to calculate:

(i) Standard deviation (ii) Variance

2. Given the data below

41	35	27	19	51	47	63	76	22	39
14	23	18	39	92	61	45	13	37	22
33	51	53	19	29	72	27	40	57	67
84	76	91	33	58	73	86	65	43	80

From a grouped frequency table with the class intervals:

11-20, 21-30, 31-40….. etc

Hence use the table to calculate:

(i) Standard deviation (ii) Variance

WEEK Eleven

11.0 SKEWNESS

A distribution is said to be symmetry if it is possible to cut its graph into two mirror image halves. Such distributions have bell-shape graphs. This shape of the frequency curves are characterized by the fact that observations equidistant from the central maximum have the same frequency e.g the normal curve. Skewness is the degree of asymmetry (departure from symmetry) of a distribution. If the frequency curve (smoothed frequency polygon) of a distribution has the length of one of its tails (relative to the central section). Disproportionate to the other, then the distribution is described as skewed. However, a skewed distribution is a distribution in which set of observations is not normally distributed that is mean, median and mode do not coincide at the middle of the curve.

The concept of skewness would be clear from the following three diagrams showing a symmetrical distribution, a positively skewed distribution and negatively skewed distribution.

1. Symmetrical Distribution

It is clear from the diagram below that in a symmetrical distribution the values of mean median and mode coincide. The spread of the frequencies is the same on both sides of the center point of the curve.

2. Asymmetrical Distribution

A distribution which is not symmetrical is called a skewed distribution and such a distribution could either be positively skewed or negatively skewed as would be clear from the following diagram

3. Positively Skewed Distribution

In the positively skewed distribution, the value of mean is maximum and that of the mode is minimum – the median lies in between the two.

4. Negatively Skewed Distribution

In a negatively skewed distribution, the value of the mode is maximum and that of mean minim um – the median lies in between the two.

Note: in moderately symmetrical distributions the interval between the mean and the median is approximately one-third of the interval between the mean and the mode. It is this relationship which provides a means of measuring the degree of skewness.

11.1 Test Of Skewness

In order to ascertain whether a distribution is skewed or not, the following tests may be applied. Skewness is present if :

The values of mean, median, and mode do not coincide.

When the data are plotted on a graph they do not give the normal bell-shape form i.e when cut along a vertical line through the centre the two halves are not equal.

Frequencies are not equally distributed at points of equal deviation from the mode.

Conversely stated, when skewness is absent, i.e in case of a symmetrical distribution, the following conditions are satisfied:

The values of mean, median, and mode coincide.

Data when plotted on a graph give the normal bell-shape form.

Frequencies are equally distributed at points of equal deviation from the mode.

However, Skewness $= \left(\dfrac{\sum (x_i - \bar{x})^3}{n-1} \right) / \sigma^3$

Example

Suppose we have the following data : 7, 9, 10, 8, 6.

$$\bar{X} = \frac{\sum x}{n}$$

X	$x - \bar{x}$	$(x-\bar{x})^2$	$(x-\bar{x})^2 / n-1$
7	-1	1	0.25
9	1	1	0.25
10	2	4	1
8	0	0	0
6	-2	4	1
Total			2.5

$\sigma^2 = \dfrac{\sum(x - \bar{x})^2}{n-1}$ and

$\sigma = \sqrt{\dfrac{\sum(x - \bar{x})^2}{n-1}}$ = 1.58

X	$x_i - \bar{x}$	$(x-\bar{x})^3$	$\dfrac{(x_i-\bar{x})^3}{n-1}$	$\left(\dfrac{(x_i-\bar{x})^3}{n-1}\right)/\sigma^3$
7	-1	-1	-0.25	-0.06
9	1	1	0.25	0.06
10	2	8	2	0.51
8	0	0	0	0
6	-2	-8	-2	-0.51
Total				0

Hence Skewness = 0

We can conclude that the distribution Symmetrical.

11.2 Exercise/Practical

The following are test scores obtained in a descriptive statistics test :

Test scores	10	20	25	30	35
Frequency	2	3	1	3	1

Calculate Skewness and comment on your result

WEEK TWELVE

12.0 Q-Q PLOT

The q-q plot is a graphical technique for determining if two data sets come from population with a common distribution. A q-q plot is a plot of the quantiles of the first data set against the quantiles of the second data set. By a quantiles, we mean the fraction (or percent) of points below the given value. That is, the 0.3 (or 30%) quantile is the point at which 30% of the data fall below and 70% fall above that value.

The advantages of the q-q plot are :

1. The sample sizes do not to be equal.
2. Many distributional aspects can be simultaneously tested. For example, shift in location, shift in scale, changes in symmetry, and the presence of outliers can all be detected from this plot. For example, if the two data come from populations whose distributions differ only by a shift in location, the should lie along a straight line that is displaced either up or down from the 45-degree reference line.

The q-q plot is similar to a probability plot. For a probability plot, the quantiles for one of the data samples are replaced with the quantiles of a theoretical distribution.

The q-q plot is formed by:

1. Vertical axis: Estimated quantiles from data set 1
2. Horizontal axis: Estimated quantiles from data set 2

Both axes are in units of their respective data sets. That is, the actual quantiles level is not plotted. For a given point on the q-q plot, we know that the quantiles is the same for both points, but not what that quantile level actually is.

The q-q plot is used to answer the following questions:

1. Do two data sets come from populations with a common distribution?
2. Do two data sets come have common location and scale?
3. Do two data sets come have similar distributional shapes?
4. Do two data sets have similar tail behavior?

Example

The data below comes from a survey of physiotherapists in Nigeria and they were asked the questions about patients who have Osteoarthritis knee. And the questions asked were

What age group are you and sex ?

For how long have you been practicing physiotherapy?

In a typical week, how many patients do you see?

On the average, about how many minutes do you spend in treating a patient ?

Create a q-q plot for years of practice and typical.

S/No	Age group	Sex	Years of practice	Typical	Ave.
1	31-40	Female	4	2	30
2	31-40	Male	14	20	45
3	21-30	Female	8	3	45
4	21-30	Male	3	5	55
5	31-40	Female	10	25	25
6	31-40	Male	10	15	30
7	31-40	Female	9	30	30
8	21-30	Female	2	150	20
9	31-40	Female	2	100	15
10	41-50	Male	17	40	45
11	21-30	Male	5	40	20
12	41-50	Male	17	15	15
13	21-30	Male	3	55	30
14	31-40	Male	11	20	20
15	31-40	Female	10	25	20
16	31-40	Male	3	15	60
17	31-40	Male	14	10	40
18	21-30	Female	2	9	45
19	51-60	Female	29	12	45
20	31-40	Male	6	10	45
21	21-30	Female	4	50	30
22	21-30	Female	5	12	35
23	21-30	Female	.	30	15
24	31-40	Female	18	50	40
25	31-40	Male	5	20	45
26	21-30	Male	5	15	30
27	31-40	Male	10	1	30
28	41-50	Male	13	10	45
29	21-30	Female	2	20	15
30	31-40	Male	7	22	30
31	31-40	Female	13	40	30
32	41-50	Male	22	40	40
33	21-30	Female	8	75	20
34	31-40	Male	9	5	20
35	31-40	Male	7	30	30
36	41-50	Male	20	13	30
37	31-40	Male	5	200	40
38	41-50	Female	24	10	20
39	31-40	Male	3	30	45
40	41-50	Female	16	30	15
41	21-30	Male	3	60	45
42	31-40	Female	11	5	20

43	31-40	Male	7	25	30
44	51-60	Male	25	3	30
45	21-30	Female	4	20	25
46	21-30	Female	3	30	30
47	31-40	Female	11	30	30
48	21-30	Male	3	4	30
49	21-30	Male	5	60	30
50	31-40	Female	16	92	60
51	21-30	Female	7	45	30
52	21-30	Male	3	10	20
53	21-30	Female	4	5	30
54	41-50	Male	16	7	30
55	31-40	Male	10	225	25
56	41-50	Male	17	40	60
57	31-40	Male	15	40	25
58	21-30	Male	2	15	40
59	21-30	Female	1	7	80
60	21-30	Female	2	2	180

Solution

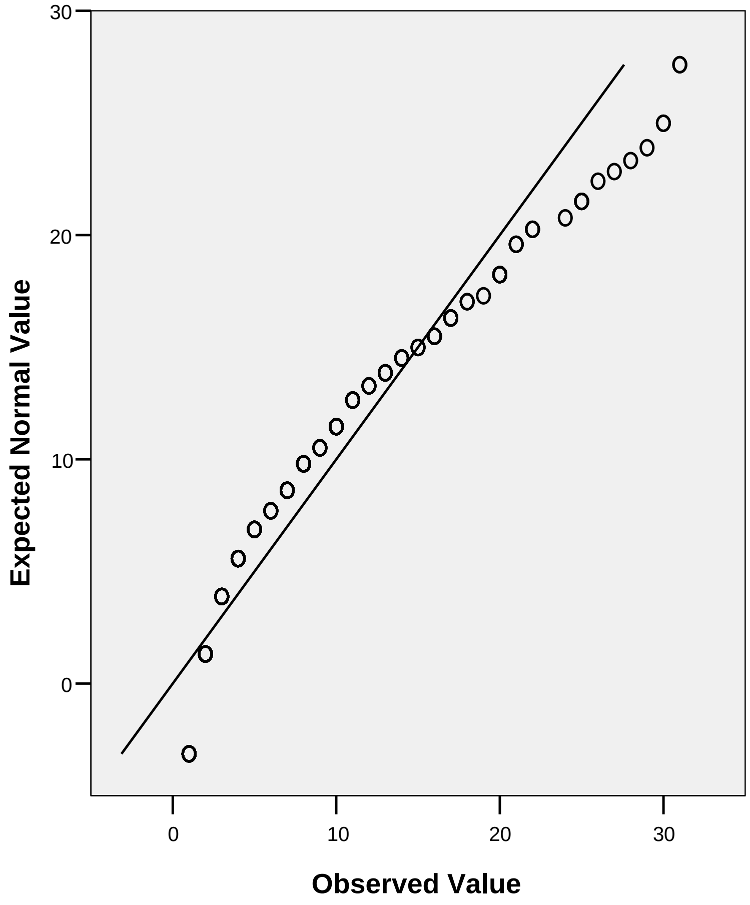
Estimated Distribution Parameters

		Years of practice as a physiotherapist	In a typical week,about how many out patient do you see
Normal Distribution	Location	8.95	37.19
	Scale	7.085	41.140

The cases are unweighted.

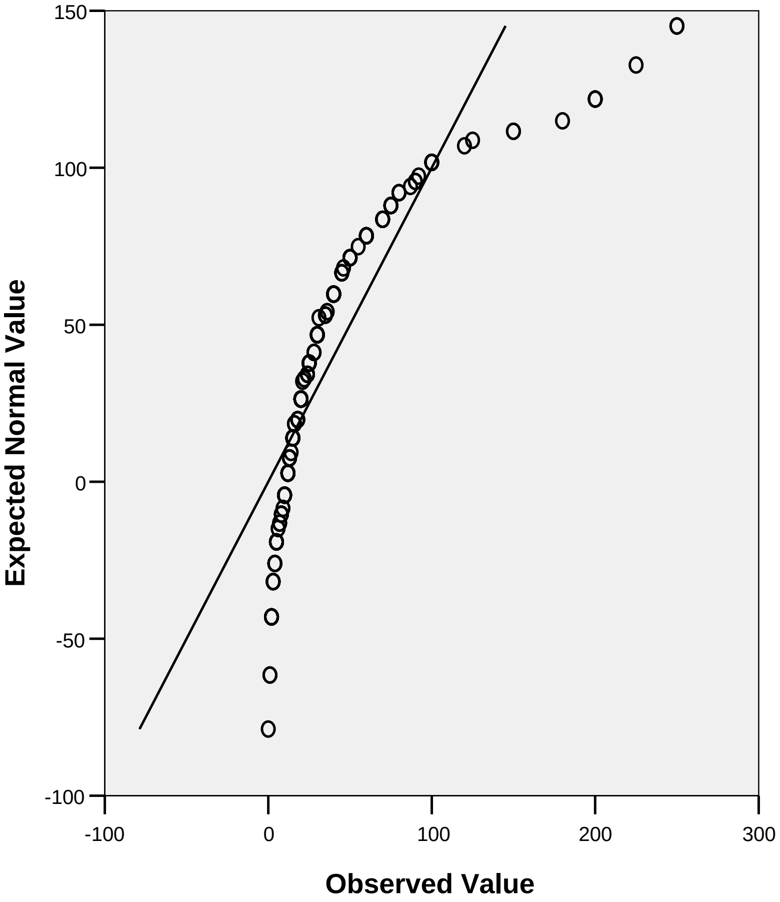
Years of practice as a physiotherapist

Normal Q-Q Plot of Years of practice as a physiotherapist



In a typical week, about how many out patient do you see

Normal Q-Q Plot of In a typical week,about how many out patient do you see



Exercise/Practical

Create a q-q plot for years of practice and average using the data in the example above.

WEEK THIRTEEN

13.0 P-P PLOT

A p-p plot also called probability-probability plot or percent-percent plot is a probability for assessing how closely two data sets agree, which plots the two cumulative distribution functions against each other.

p-p plot are sometimes limited to comparisons between two samples, rather than comparison of a sample to a theoretical model distribution. However, they are of general use, particularly where observations are not all modeled with the same distribution.

However, they have found some use in comparing a sample distribution from known theoretical: given n samples, plotting the continuous theoretical cumulative distribution function against the empirical cumulative distribution function would yield a stair-step

Example

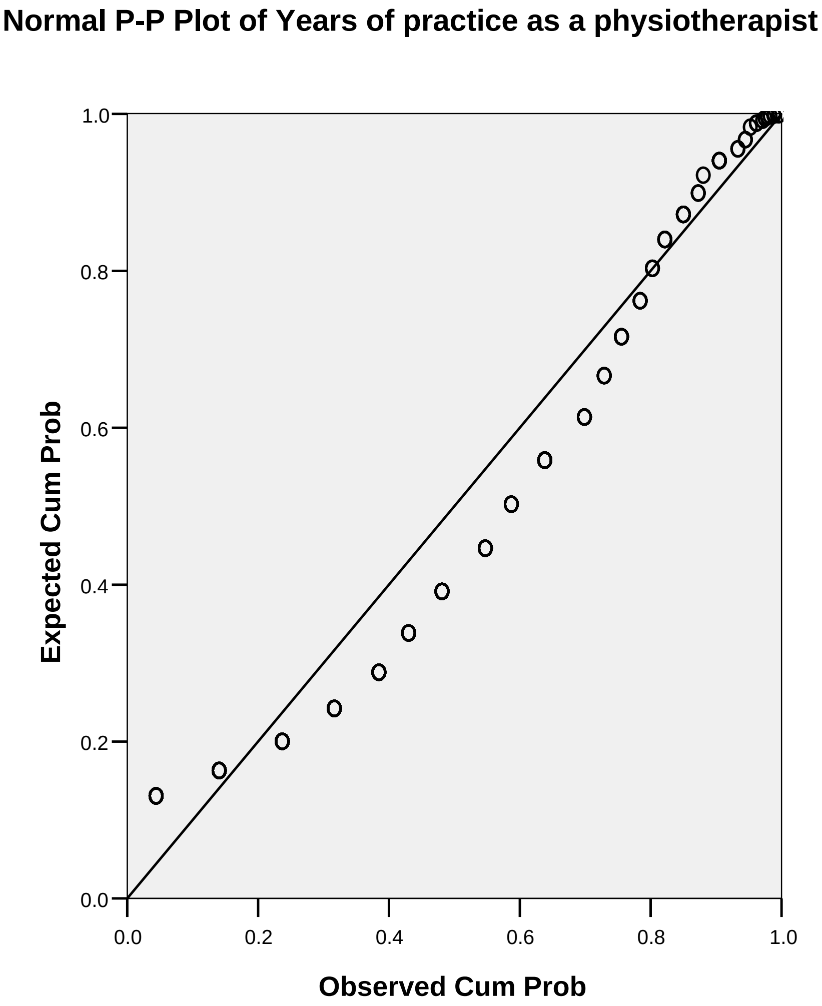
Consider the data in twelve.

Estimated Distribution Parameters

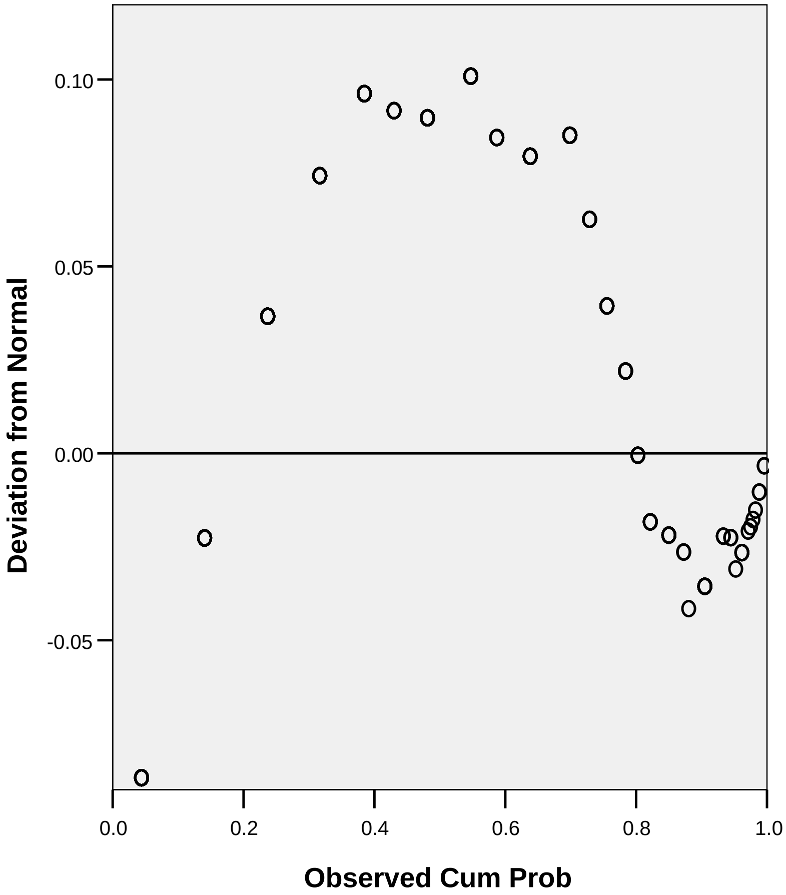
		Years of practice as a physiotherapist	On the average, about how many minutes do you spend treating Osteoarthritis knee (Initial visit)
Normal Distribution	Location	8.95	47.43
	Scale	7.085	20.146

The cases are unweighted.

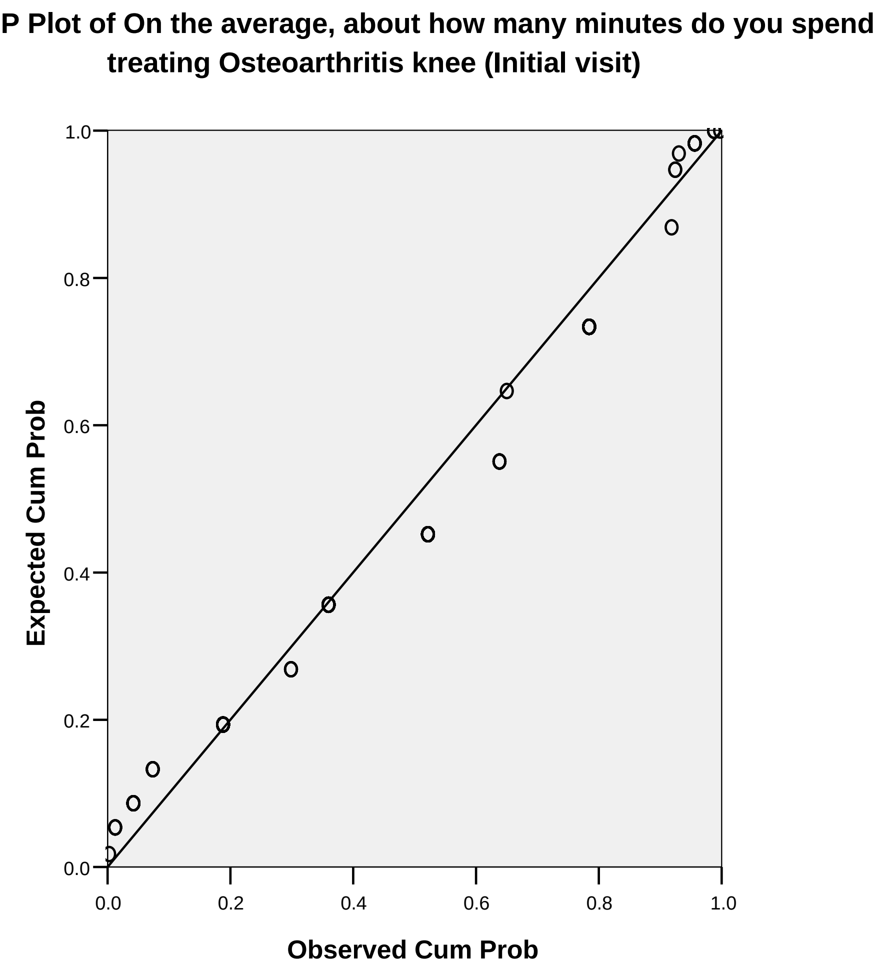
Years of practice as a physioterapist



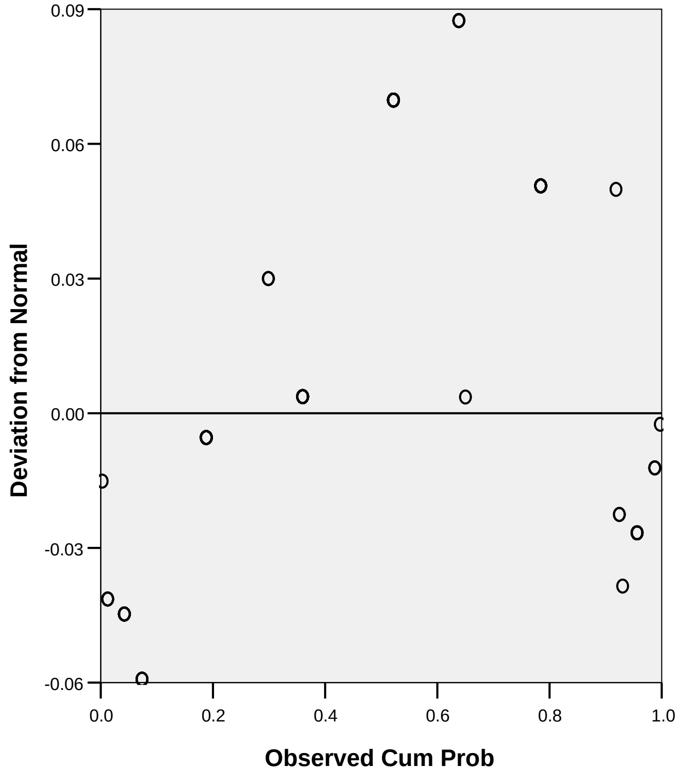
Detrended Normal P-P Plot of Years of practice as a physiotherapist



On the average, about how many minutes do you spend treating Osteoarthritis knee (Initial visit)



Detrended Normal P-P Plot of On the average, about how many minutes do you spend treating Osteoarthritis knee (Initial visit)



Exercise/Practical

The data below comes from a survey of physiotherapists in Nigeria and they were asked the questions about patients who have Osteoarthritis knee. And the questions asked were

What age group are you and sex ?

For how long have you been practicing physiotherapy?

In a typical week, how many patients do you see?

On the average, about how many minutes do you spend in treating a patient ?

Create a P-P plot in a typical week, how many patients do you see? (typical).

S/No	Age group	Sex	Years of practice	Typical	Ave.
1	31-40	Female	4	2	30
2	31-40	Male	14	20	45
3	21-30	Female	8	3	45
4	21-30	Male	3	5	55
5	31-40	Female	10	25	25
6	31-40	Male	10	15	30
7	31-40	Female	9	30	30
8	21-30	Female	2	150	20
9	31-40	Female	2	100	15
10	41-50	Male	17	40	45
11	21-30	Male	5	40	20
12	41-50	Male	17	15	15
13	21-30	Male	3	55	30
14	31-40	Male	11	20	20
15	31-40	Female	10	25	20
16	31-40	Male	3	15	60
17	31-40	Male	14	10	40
18	21-30	Female	2	9	45
19	51-60	Female	29	12	45
20	31-40	Male	6	10	45
21	21-30	Female	4	50	30
22	21-30	Female	5	12	35
23	21-30	Female	.	30	15
24	31-40	Female	18	50	40
25	31-40	Male	5	20	45
26	21-30	Male	5	15	30
27	31-40	Male	10	1	30
28	41-50	Male	13	10	45

29	21-30	Female	2	20	15
30	31-40	Male	7	22	30
31	31-40	Female	13	40	30
32	41-50	Male	22	40	40
33	21-30	Female	8	75	20
34	31-40	Male	9	5	20
35	31-40	Male	7	30	30
36	41-50	Male	20	13	30
37	31-40	Male	5	200	40
38	41-50	Female	24	10	20
39	31-40	Male	3	30	45
40	41-50	Female	16	30	15
41	21-30	Male	3	60	45
42	31-40	Female	11	5	20
43	31-40	Male	7	25	30
44	51-60	Male	25	3	30
45	21-30	Female	4	20	25
46	21-30	Female	3	30	30
47	31-40	Female	11	30	30
48	21-30	Male	3	4	30
49	21-30	Male	5	60	30
50	31-40	Female	16	92	60
51	21-30	Female	7	45	30
52	21-30	Male	3	10	20
53	21-30	Female	4	5	30
54	41-50	Male	16	7	30
55	31-40	Male	10	225	25
56	41-50	Male	17	40	60
57	31-40	Male	15	40	25
58	21-30	Male	2	15	40
59	21-30	Female	1	7	80
60	21-30	Female	2	2	180

WEEK Fourteen

14.0 PROBABILITY AND NON-PROBABILITY METHODS

The various methods of sampling can be grouped under two broad heads:

Probability sampling and

Non-probability sampling

14.1 Probability sampling methods are those in which every item in the population has a known probability of being chosen. This implies that the selection of sample items is independent of the person making the study.

Probability sampling methods are as follows:

1. Simple random sampling
2. Stratified sampling
3. Systematic sampling
4. Cluster sampling

Simple random sampling:-This is the simplest sampling design defined as the sampling design in which every unit in the population has the same probability 1/N of being selected at each draw. The probability of a unit been selected into sample is also for each unit namely n/N of appearing in the sample.

N being the size of the population and n that of sample, Random number table can be reached by a column or row provided the method is consistent.

Selection procedure:

a. Lottery
b. Use of table of random numbers

Advantages

It is an equitable method of selection i.e. no personal bias

The precision of estimate is the highest of all types of probability sampling methods.

Disadvantages

Without sample frame, selection cannot be done.

If the population is larger, it is time consuming, labourious and expensive.

Stratified sampling:- This is a procedure of sampling in which the population to be sampled is first divided into sub population called strata (stratum for singular) which are as much internally homogenous as possible with respect to the variable under study. Selecting a random sample from each sub population independently according to some criteria. For instance, selecting n from N objects. The population can be divided first into strata and then samplings taking from each stratum. The stratification may be only on one variety as sex or may be more complex e.g. males, females, ages, social classes, ethnicity area, occupation and education are often used as a variable for strata.

Advantages

More representative

Increase in precision of estimate.

Provision for reasonable accuracy.

More useful in heterogeneous population.

The sub population totals are known.

Disadvantages

It needs highly trained personnel.

Labourious in preparation of sample frame.

May be costly if stratified sampling frame are not available.

Systematic sampling:- This is the method of sampling in which the first unit sample is selected using random number and the remaining units are automatically selected by a pre determined rule.

Selection of systematic random sample involves to the following steps:-

i. Taking population size. N= 30

Sample size n = 10

Sampling fraction = n/N = 10/30 = 1/3

Sampling interval = N/n =30/10 = 3

ii. Select a random start, r(1 ≤r ≤K) i.e. which will be the first selected unit in the list of sampling unit

iii. The other units are selected by successively adding K to the random start.

Advantages

Easy to select

Preparation of sample frame is not necessary

The sample is spread evenly over the entire population

When there are not periodic event that are associated with sampling intervals, the result obtained they are satisfactory.

Saves time

Disadvantage

If the sampling intervals coincide with any periodic interval, the result will not be a good representation.

Randomness is not maintained, i.e. only the items within the range of the first selection are given equal chance of selection.

Cluster sampling:- This is a sampling method where the population is made up of groups of element like in the case of stratification only that the groups in this case comprises of heterogeneous elements (i.e. different kind of elements). Each and in the selection procedure a probability selection procedures is applied to select groups (cluster) from all the groups that make up the population and study is then been conducted as the selected cluster giving attentions to all the elements in the selected cluster.

Advantages

Less field cost is required.

When there is no satisfactory sampling frame, this method can be used.

Disadvantages

If unit in the cluster are similar or correlated it is waste of resources.

Non-probability sampling.

There is no way of estimating the probability of each member of the population being included in the study. The following are types of Non-probability sampling:-

i. Quota sampling
ii. Accidental sampling
iii. Purposive/Judgmental sampling

i. *Quota sampling: -* In this technique, the principles which applies are similar to that of stratify, the difference is not random. Hence the knowledge of strata of the population such as sex, age, education, etc are used to select sample element that will be representative i.e Quota or proportion is assigned to elements of the population.

Advantages

Reduces cost of selecting sample.

Introduces some stratification effect

Disadvantages

Introduces bias of the observers classification of units.

Introduces bias due to non random selection of units.

ii. *Accidental sampling:-* Sometimes it might be difficult for a researcher to sample on random basis on the element of the population, this is because some element on random might not be reached or available on one reason or the other. An accidental sampling can be adopted where a researcher can decide, for example the first ten (10) people he meets on entering an organization will be interviewed for his/her research.

iii. *Purposive/Judgmental sampling*:- In this type of sampling, the selection procedure is done on the expert devices or some reason which is used as a basis for judging the inclusion of an element in the sample e.g. a survey of industry. The sample is selected for study is based on the rate of response of such industry in the past such selection procedure is known as “judgmental”.

Advantages

Requires strong assumptions or knowledge of population and sub group selected.

Variability and bias of estimates cannot be measured or controlled.

Exercise

Distinguish between the two sources of data collection

WEEK Fifteen

15.0 Method of data collection

Data are generally and collected to provide useful and meaningful information about the observation under study. The entire planning and execution a survey depend on data availability which is greatly influenced by the method of data collection. Decision and choice of the method of the collection should be arrived after careful consideration of aims and objectives of the survey. The nature of in formation is needed, the population under study degree of accuracy, practically, time and cost. The following are method of data collection:-

1. Questionnaire method
2. Interview method
3. Observation method
4. Documentary method

15.1 Questionnaire method

This method involves the use of questionnaire (statistical format) to collect the needed statistical information. Questionnaire are specially designed forms meant to extract available information from respondent (i.e. persons, group of persons, organization, or institutions) Questionnaire can be comprises of different logical arranged question which are supposed to be answer by respondent. After answering the question, the respondent is expected to send the format back to the source. This method is widely and commonly used in collecting information because of certain advantages it has.

Advantages

a. Wide coverage - Questionnaire can be distributed to a large number of individuals, groups or institutions etc.

b. Saves time - Questionnaire can be distributed, filled and return back to the source within a very short time.

c. Less cost- Money cost incurred by using a questionnaire to collect information is relatively small e.g. you send questionnaire to the institute in London and get the questionnaire back with little amount of money.

d. Different types of information can be collected by using questionnaire method.

Disadvantages

a. Problem of designing questionnaire itself. Care should be taken in its design to remove ambiquility, repetition and out of point questions.

b. Non-response- There is tendency that the respondent may not respond to a question due to low level of awareness or knowledge of the importance of such questionnaire.

c. Incomplete or Inaccurate- There is tendency that the respondent may not complete the questionnaire or give accurate answers to the questions.

d. Wrong or False information- The respondents may deliberately give wrong /false information without the investigator knowledge.

15.2 Interview Method

The method involve a personal contact of the interviewer with the respondent (interview) during which the interviewer ask the respondent a series of questions concerning the subject matter and the respondent is expected to answer. It is an oral interaction between the two parties. Interview can both be face to face question and answer through or through the telephone.

Advantages

a. It allows free face to face interaction between the interviewer and the respondent

b. It allows more detailed information to be collected with full explanation

c. It allows the interviewer to guides or directs the respondent accurately in completing in formations.

d. False information can be checked or corrected when noticed by the interviewer.

Disadvantages

a. Only small areas is covered because it is not possible to interview a larger number of persons.

b. Time wasting when covering a large number of individuals

   High cost; since the interviewer has to follow the respondent on after the other to respective destination

c. Appointment should be booked in advance with the respondent before conducting the interview

d. Respondent personal feelings may influence the accuracy of the information given

15.3 Observation method

This is method of systematic and scientific enquiry used to collect data in almost all disciplines and for controlled experiment such as biological, social ,economic and physical laboratory experiments. it is on –the- spot watch of an event taking place or happening.It is rampantly used in experiment because of its high degree of accuracy and efficiency in providing the needed information.

Advantages

a. Information collected are directly from the life happening since events are recorded as they happens

b. There is contact between the observer and the subject matter being observed

c. The observation can guide control and influence the process to obtain the most accurate information

d. The information recorded through this method is highly rehired since the recorded information is what the observers has seen with his/her own eyes.

Disadvantages

a. Time consuming-since the observer has to wait for the events or process to start and end

b. High cost- if the money is proportional to the consumed

c. There is problem of maintaining the object to be studied or observed and setting the experiment in notion.

15.4 Documentary methods

The used of the already existing record (documents) of past / present predicted future events or phenomena is known as documentary method of data collect. Documents may either be official or unofficial. in this situation, if information is collected from an official document then the data is referred to as official and if otherwise is unofficial. This method makes use of books, newspaper, report, bulleting library from official sources

Advantages

a. Save time; information can extracted from an existing document within a short time.

b. Provide old information about an event happening can be recorded.

c. Less labor input involved is collecting information through documents

d. Less cost; money cost incurred by the method is small

e. Provide official information

Disadvantages

a. If the information collected was from initial stage wrong, inaccurate or incomplete information will be collected.

b. If false official and documented information is recorded in a document, there is practically no room to correct such information

Exercise/Practical

1. Students should go to various institutions (hospital, market, ministries etc) and collect data on various activities.
2. Discuss the various data collection method described in this section.