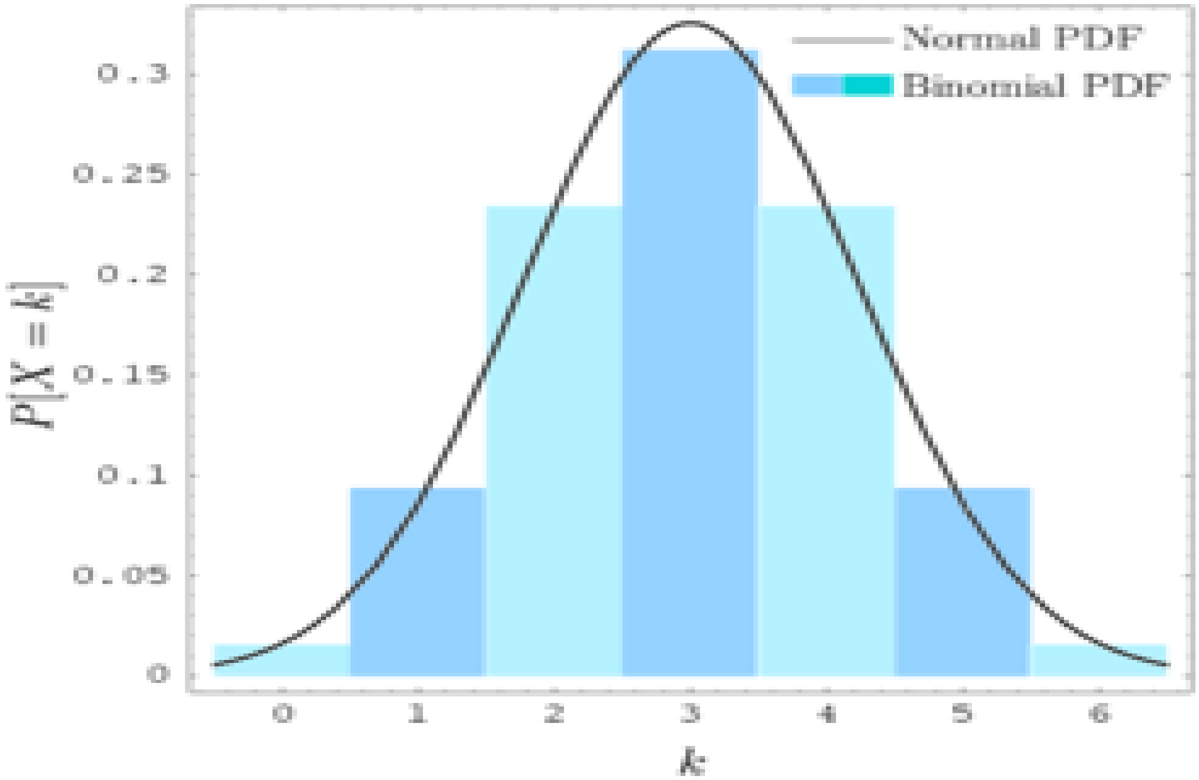
UNESCO-NIGERIA TECHNICAL & VOCATIONAL EDUCATION REVITALISATION PROJECT-PHASE II

NATIONAL DIPLOMA IN STATISTICS



INTRODUCTION TO PROBABILITY

COURSE CODE: STA 112

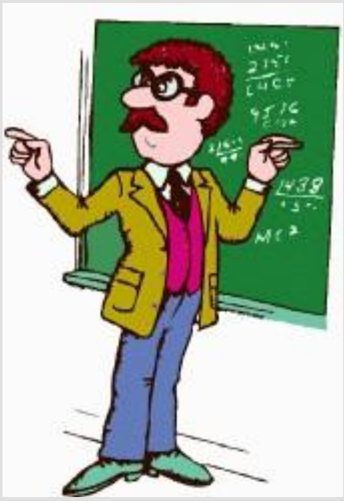
YEAR I- SEMESTER I

THEORY/PRACTICAL

Version 1: July 2009

CONTENTS

WEEK 1



Introduction to Probability

Probability and Statistics are related in an important way. Probability is used as a tool; it allows you to evaluate the reliability of your conclusions about the population when you have only sample information. In a nutshell, the probability of an event A is a measure of our belief that the event A will occur. One practical way to interpret this measure is with the concept of relative frequency. The probability of an event is simply its long-run relative frequency.

In a class of 25 students, each asked to identify the month and day of his or her birth. What are the chances that at least two students will share the same birthday? Our intuition is misleading; it happens that at least two students will have the same birthday in more than half of all classes with 25 students.

Week 1

In Nigeria, out of the 45 million voters, a pollster needs to survey only 2000 (or 0.001%) in order to get a good estimate of the number of voters to favour a particular candidate.

The preceding conclusions are based on simple principles of probability which play a critical role in the theory of statistics. All of us now form simple probability conclusions in our daily lives. Sometimes these determinations are based on fact, while others are subjective. In addition to its importance in the study of statistics, probability theory is playing an increasingly important role in a society that must attempt to measure uncertainties. Before arming a nuclear warhead, we should have some knowledge about the probability of an accidental detonation. Before raising the speed limit on our nation's highways, we should have some knowledge of the probability of increased fatalities.

Subsequent chapters will develop methods and skills that will enable us to calculate probabilities easily. It is therefore important to acquire a basic understanding of probability theory. We want to cultivate some very basic skills in calculating the probabilities of certain events.

Example

In a population of a town with 140,000 educated and 60,000 uneducated people, we can see that if one person is randomly selected, there are 60,000 chances out of 200,000 of picking an uneducated person. This corresponds to a probability of 60,000/200,000 or

0.3. The population of all people in the town is known, and we are concerned with the likelihood of obtaining a particular sample (an uneducated person). We are making a conclusion about a sample based on our knowledge of the population. The knowledge of sets, permutations and combinations are essential to understanding both the theory and applications of probability.

1.1 Introduction to Sets with Examples

A set is a collection of objects. The objects in a set are called elements of the set. Sets are indicated by means of braces, $\{\ \}$, and are often named with capital letters and the elements denoted by small letters. Sets are used in many areas of mathematics, so an understanding of sets and set notation is important. When the elements of a set are listed within the braces, as illustrated below, the set is said to be in ***roster form***.

$A = \{a, b, c\}$

$B = \{1,2,3,4,5\}$

$C = \{Abia, Bauchi, Kano, Oyo\}$

Set A has three elements, set B has five elements and set C has four elements. Since 2 is an element of set B, we may write $2 \in B$; this is read “2 is an element of set B”.

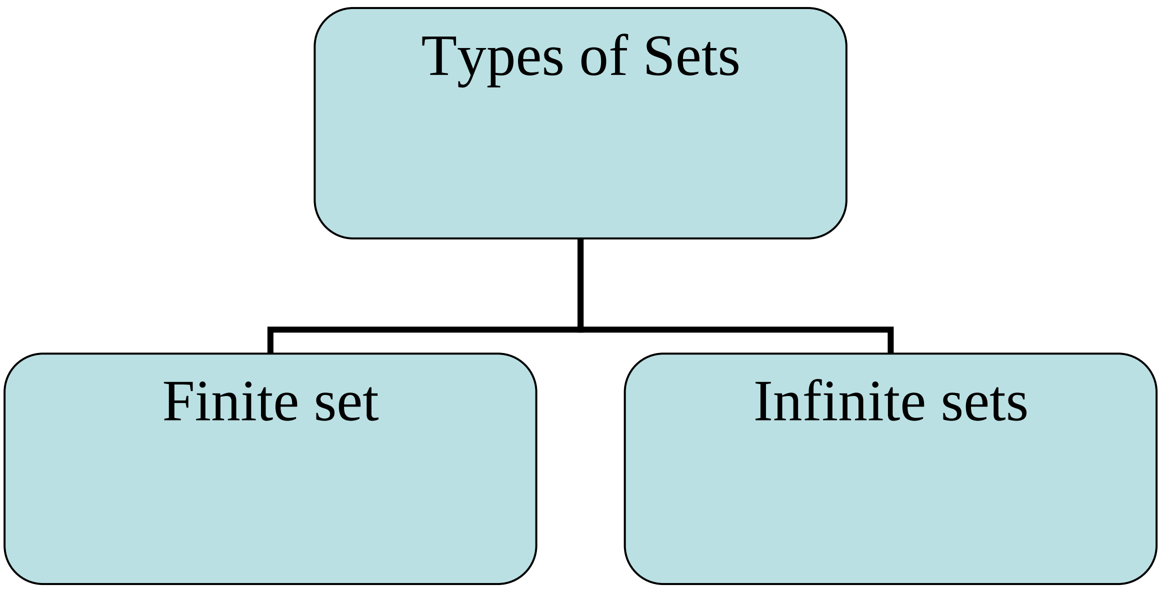
A set may be finite or infinite. Set A, B, and C each has a finite number of elements and all are therefore finite sets. In some sets, it is impossible to list all the elements. These are infinite sets. The following set, called the set of natural numbers or counting numbers, is an example of an infinite set.

$N = \{1,2,3,4,5,...\}$

The three dots after the last comma, called an ***ellipsis***, indicate that the set continues in the same manner. Another important infinite set is the set of integers which follows.

$I = \{... -4,-3,-2,-1,0,1,2,3,4,...\}$

It is important to note that the set of integers include both positive and negative integers and the number 0.



If we write $D = \{1,2,3,4,5,...,250\}$, we mean that the set continues in the same manner until the number 250. Set D is the set of the first 250 natural numbers which is therefore a finite set.

A special set that contains no element is called a null set, or empty set, written $\{\ \}$ or Ø. For example, the set of students in your class over the age of 150 is a null or empty set.

Another method of indicating a set, called ***set builder notation***. An example of set builder notation is:

$E = \{x / x \text{ is a natural number greater than } 6\}$

This is read "set *E* is the set of all elements *x*, such that *x* is a natural number greater than 6". In roster form, this is written as:

$E = \{7,8,9,10,11,...\}$

Two condensed ways of writing set

$E = \{x / x \text{ is a natural number greater than } 6\}$ in set builder notation follow:

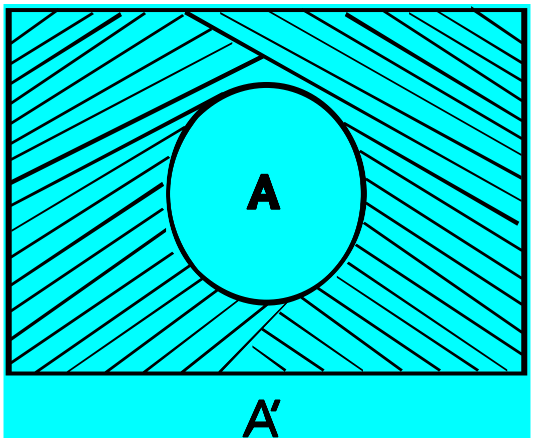
$E = \{x / x > 6 \text{ and } \quad x \in N\}$

or $E = \{x / x \geq 7 \text{ and } \quad x \in N\}$

The set $A = \{x / -3 < x \leq 4 \text{ and } \quad x \in I\}$ is the set of integers greater than -3 and less than or equal to 4. The set written in roster form is $A = \{-2,-1,0,1,2,3,4\}$. Notice that the endpoint -3 is not included in the set but the endpoint 4 is included.

1.2 Universal Set and Subset

A ***universal set***, usually denoted by ε or U, is a master set in which some subsets are defined under it. If A is a subset of a universal set U, the ***complement*** of A, denoted by A', is the set of elements which belong to U but do not belong to A. Hence, *A'* is shaded in the following diagram.



Examples

1. If $U = \{1,2,3,4,5,6,7,8\}$ and $A = \{3,4,5,6,7\}$

Then; $A' = \{1,2,8,\}$

2. If $U = \{a,b,c,d,e,f\}$, $X = \{a,b,c\}$ and $Y = \{c,d,e\}$, find $X \cap Y'$

Solution; here we need to find $Y' = \{a,b,f\}$.

Then $X \cap Y' = \{a,b\}$

Important Definitions

Equal sets

Two or more sets are equal if and only if they contain exactly the same elements regardless of the arrangements of the elements. For example sets $A = \{x, y, z\}$ and $B = \{z, y, x\}$ are equal. That is, $A = B$.

Equivalent sets

Two or more sets are equivalent if and only if they contain exactly the same number of elements. Here we are only talking in terms of the number of elements but not the actual elements. For example sets $A = \{x, y, z\}$ and $B = \{1,2,3\}$ are equivalent. That is, $A \equiv B$.

Cardinal number of a set

The cardinal number of a set or simply the cardinality of a set is the number of elements in that particular set. The cardinal number of a set A is denoted by $n(A)$. For example, the cardinal number of the sets $A = \{x, y, z\}$ and $B = \{\ \}$ are respectively given as $n(A) = 3$ and $n(B) = 0$. The cardinality of a set is closely related to the probability theory as we shall see later.

1.3 Elements of a Set

The concept of the elements of a set is a simple; we shall use the following examples to illustrate this concept as follows:

$A = \{1,3,5\}$

$B = \{a, b, c, d, e\}$

$C = \{Adamawa, Benue, Kwara, Ogun\}$

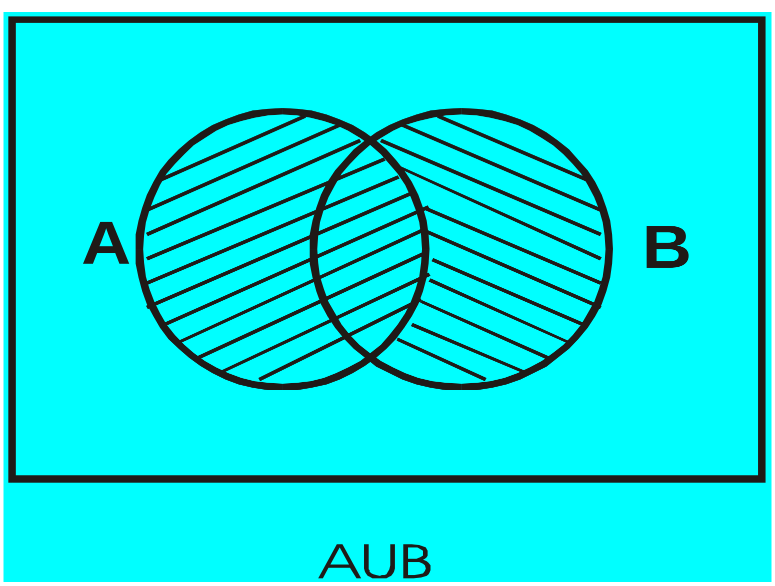
Set A has three elements, set B has five elements and set C has four elements. Since 2 is an element of set B, we may write $2 \in B$; this is read "2 is an element of set B".

1.4 Notations of a Set

Just as operations such as addition and multiplication are performed on numbers, operations can be performed on sets. Two operations are union and intersection.

Union of sets

The union of set A and set B, written $A \cup B$, is the set of elements that belong to either set A or set B. in other words, it is the combination of set A and set B without repetition of elements. The shaded portion in the diagram below represents $A \cup B$.



Examples

1. If $A = \{1,2,3,4,5\}$ and $B = \{3,4,5,6,7\}$

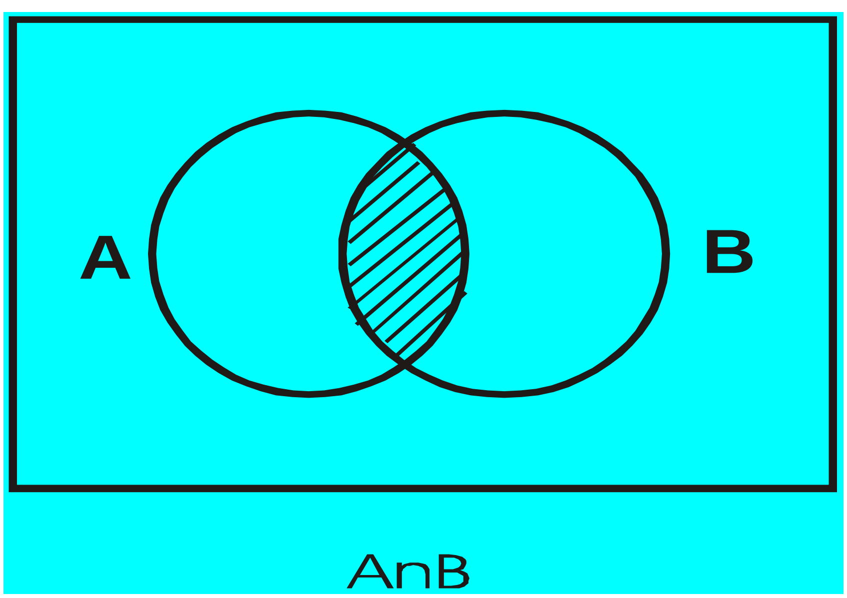
Then; $A \cup B = \{1,2,3,4,5,6,7\}$

2. If $A = \{a,b,c,d,e\}$ and $B = \{x,y,z\}$

Then; $A \cup B = \{a,b,c,d,e,x,y,z\}$

Intersection of sets

The intersection of set A and set B, written $A \cap B$, is the set of all elements that are common to both set A and set B. the intersection is formed by using only those elements that are in both set A and set B. if an item is an element in only one of the two sets, then it is not an element in the intersection of the sets. The blackened portion in the diagram below represents $A \cap B$.



Examples

1. If $A = \{1,2,3,4,5\}$ and $B = \{3,4,5,6,7\}$

Then; $A \cap B = \{3,4,5,\}$

2. If $A = \{a, b, c, d, e\}$ and $B = \{x, y, z\}$

Then; $A \cap B = \{\ \}$

Note that in the last example, set A and set B have no elements in common. Therefore, their intersection is the empty set.

Week 1 Practical Activities

Application of Sets in Economics

The table below gives the rate of inflation, as measured by the percentage change in the consumer price index (CPI), for the years 1988 to 2006. Let

$A = \{years\ \ from\ 1988\ to\ 2006\ in\ which\ \text{inflation}\ was\ above\ 10\%\}$

$B = \{years\ \ from\, 1988\ to\ 2006\ in\ which\ \text{inflation}\ was\ below\ 4\%\}$

Determine the elements of A and B and give some economic interpretation to each of them.

Years	Inflation	Years	Inflation	Years	Inflation
1988	7.7	1995	3.6	2002	3.0
1989	11.3	1996	1.9	2003	2.6
1990	13.5	1997	3.6	2004	2.6
1991	10.4	1998	4.1	2005	2.5
1992	6.1	1999	4.8	2006	3.4
1993	3.2	2000	5.4		
1994	4.3	2001	4.2		

We see that:

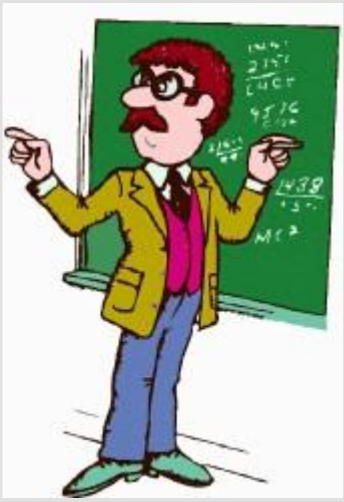
$A = \{1989, 1990, 1991\}$

$B = \{1993, 1995, 1996, 1997, 2002, 2003, 2004, 2005, 2006\}$

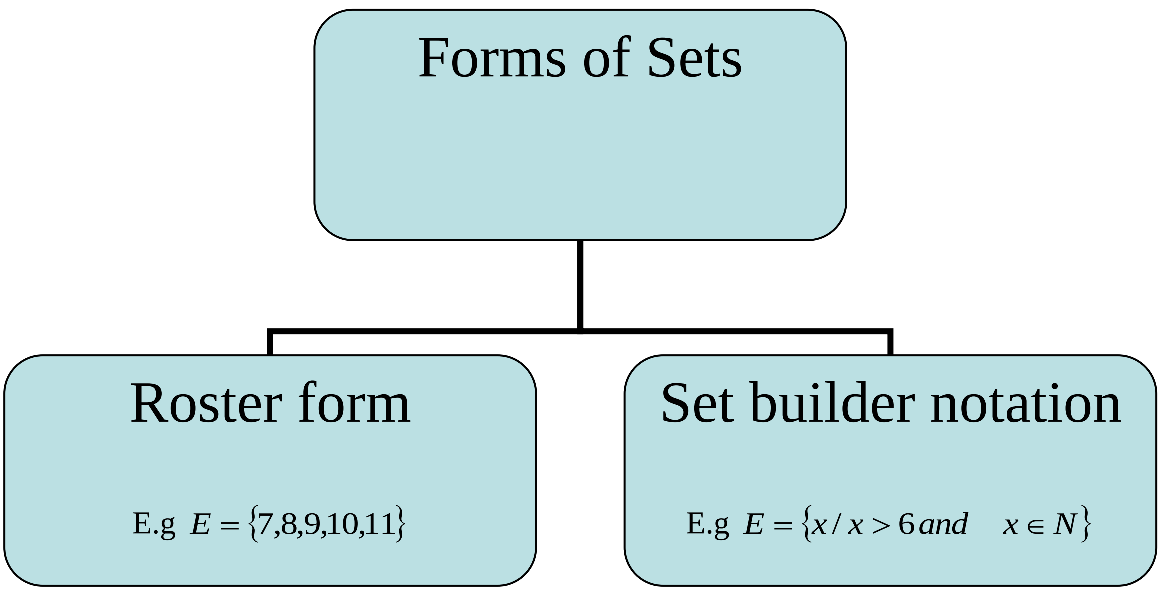
Students are expected to visit the National Bureau of Statistics (NBS) or the Central Bank of Nigeria (CBN) and collect similar data for at least 20 years to make similar set classifications with appropriate interpretations.

Week 1

WEEK 2



1.5 Roster and Set Builder Forms of Sets



Sets are used in many areas of mathematics and statistics, so an understanding of sets and set notation is important. When the elements of a set are listed within the braces, as illustrated below, the set is said to be in ***roster form***.

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$N = \{1,2,3,4,5,...\}$

Week 2

The three dots after the last comma, called an ***ellipsis***, indicate that the set continues in the same manner. Another important infinite set is the set of integers which follows.

$I = \{\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

It is important note that the set of integers include both positive and negative integers and the number 0.

If we write $D = \{1, 2, 3, 4, 5, \ldots, 250\}$

We mean that the set continues in the same manner until the number 250. Set D is the set of the first 250 natural numbers which is therefore a finite set.

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1.6 Null Set

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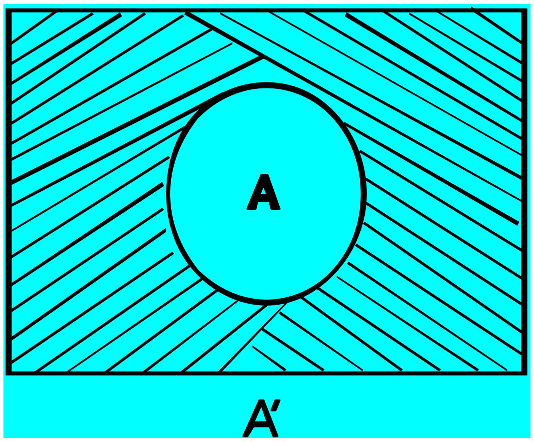
1.7 Universal Set

A ***universal set***, usually denoted by ε or U, is a master set in which some subsets are defined under it. If A is a subset of a universal set U, the ***complement*** of A, denoted by A', is the set of elements which belong to U but do not belong to A. Hence, A' is shaded in the following diagram.



Examples

1. If $U=\{1,2,3,4,5,6,7,8\}$ and $A=\{3,4,5,6,7\}$

Then; $A'=\{1,2,8,\}$

2. If $U=\{a,b,c,d,e,f\}$, $X=\{a,b,c\}$ and $Y=\{c,d,e\}$, find $X\cap Y'$

Solution; here we need to find $Y'=\{a,b,f\}$.

Then $X\cap Y'=\{a,b\}$

Exercise

1. Given $P=\{1,2,3,4\}$ and $Q=\{3,5,6\}$, find:

(a) $P \cap Q$,

(b) $P \cup Q$,

(c) $(P \cap Q) \cup Q$

2. If $P = \{2,1,3,9,7\}, Q = \{1,8,3,7\}$ and $R = \{5,4,8\}$, find:

(a) $P \cup Q \cup R$,

(b) $P \cap Q \cap R$

3. Given that $S = \{1,2,3,4,5,6\}, T = \{2,4,5,7\}$ and $R = \{1,4,5,\}$, find $(S \cap T) \cup R$.

4. If $P = \{2,1,3,9,4\}, Q = \{1,5,3,7\}$ and $R = \{5,4,6,1\}$, find:

(a) $P \cup Q \cup R$,

(b) $P \cap Q \cap R$

Answers

1. (a) $P \cap Q = \{3\}$

(b) $P \cup Q = \{1,2,3,4,5,6\}$

(c) $(P \cap Q) \cup Q = \{3,5,6\}$

2. (a) $P \cup Q \cup R = \{1,2,3,4,5,7,8,9\}$

(b) $P \cap Q \cap R = \{\ \}$

3. $(S \cap T) \cup R = \{1,2,4,5\}$

4. (a) $P \cup Q \cup R = \{1,2,3,4,5,6,7,9\}$

(b) $P \cap Q \cap R = \{1\}$

Week 2

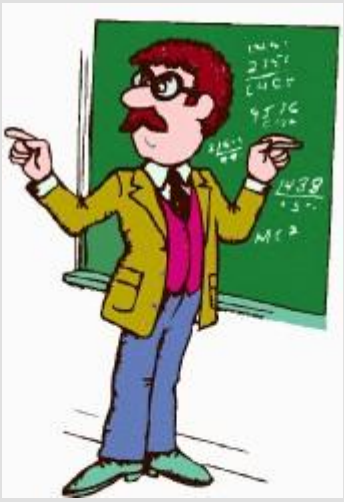
Week 2 Practical Activities

The Application of Sets to Business

A survey of the characteristics of 100 small businesses that had failed in Nigeria revealed that 95 of them either were undercapitalized, had inexperienced management or had a poor location. Four of the businesses had all three of these characteristics. Forty businesses were undercapitalized but had experienced management and good location. Fifteen businesses had inexperienced management but sufficient capitalization and good location. Seven were undercapitalized and inexperienced management. Nine were undercapitalized and had poor location. How many of the businesses had poor location? Which of the three characteristics was most prevalent in the failed businesses?

NB: Students are expected to visit the National Bureau of Statistics (NBS) or the Central Bank of Nigeria (CBN) and collect similar data on failed banks in Nigeria and use sets make similar analysis.

WEEK 3



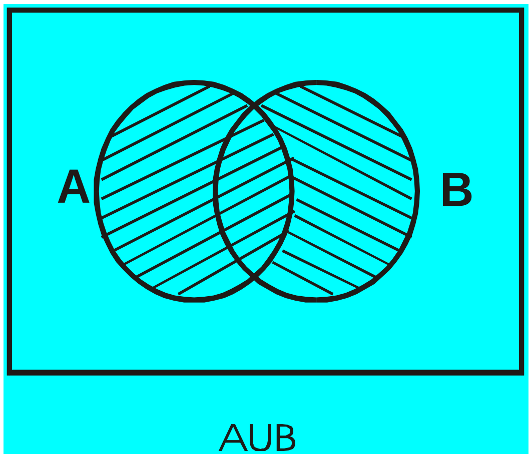
1.8 Probability and Set Operations

The probability of an event is its long-run relative frequency. Just as operations such as addition and multiplication are performed on numbers, operations can be performed on sets. Two operations are union and intersection.

Often, instead of individual possibilities, we want to talk about combinations of outcomes. These combinations of possibilities are better understood through the union and intersection of sets.

Union of sets

The union of set A and set B, written $A \cup B$, is the set of elements that belong to either set A or set B. in other words, it is the combination of set A and set B without repetition of elements. The shaded portion in the diagram below represents $A \cup B$.



Examples

1. If $A = \{1,2,3,4,5\}$ and $B = \{3,4,5,6,7\}$

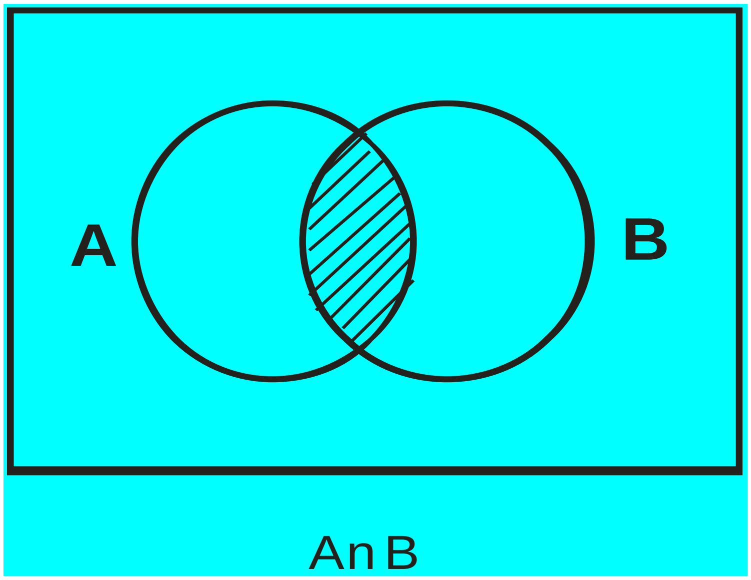
Then; $A \cup B = \{1,2,3,4,5,6,7\}$

2. If $A = \{a, b, c, d, e\}$ and $B = \{x, y, z\}$

Then; $A \cup B = \{a, b, c, d, e, x, y, z\}$

<u>Intersection of sets</u>

The intersection of set A and set B, written $A \cap B$, is the set of all elements that are common to both set A and set B. the intersection is formed by using only those elements that are in both set A and set B. if an item is an element in only one of the two sets, then it is not an element in the intersection of the sets. The blackened portion in the diagram below represents $A \cap B$.



Examples

1. If $A = \{1,2,3,4,5\}$ and $B = \{3,4,5,6,7\}$

Then; $A \cap B = \{3,4,5,\}$

2. If $A = \{a, b, c, d, e\}$ and $B = \{x, y, z\}$

Then; $A \cap B = \{\ \}$

Note that in the last example, set A and set B have no elements in common. Therefore, their intersection is the empty set.

1.9 Laws of Sets as Applied to Probability

Commutative Laws:

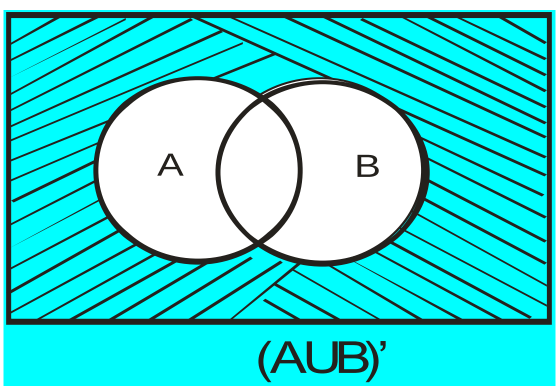
1. $A \cup B = B \cup A$

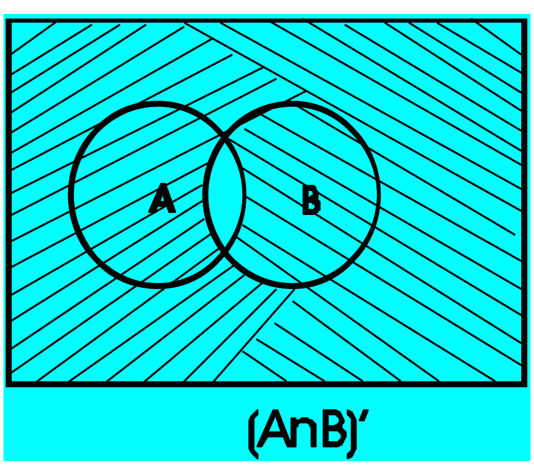
2. $A \cap B = B \cap A$
Associative Laws: 1. $(A \cup B) \cup C = A \cup (B \cup C)$ 2. $(A \cap B) \cap C = A \cap (B \cap C)$
Distributive Laws: 1. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ 2. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Idempotent Laws: 1. $A \cup A = A$ 2. $A \cap A = A$
Identity Laws: 1. $A \cup \{\ \} = A$ 2. $A \cup \varepsilon = \varepsilon$ 3. $A \cap \{\ \} = \{\ \}$ 4. $A \cap \varepsilon = A$
Complement Laws: 1. $A \cup A' = \varepsilon$ 2. $A \cap A' = \{\ \}$ 3. $(A')' = A$
De Morgan's Laws: 1. $(A \cup B)' = A' \cap B'$ 2. $(A \cap B)' = A' \cup B'$

Week 3

1.10 Probability and Venn Diagram

Venn diagram helps to better explain the concepts of compound probability. The schematic representation of a set was first used in the 19th century by the English Mathematician, John Venn. A universal set U will typically be represented by a rectangle, and the subsets of U by the interior of a circle lying wholly within the rectangle. The Venn diagram is an easy and practical way of representing sets. In most practical situations, the cardinality of the sets is used as against the actual elements of the sets. This humble contribution has further enhanced the theory of probability. The Venn diagram has made the application of set theory more practical than ever because solutions to real-life problems are obtained through it. Some examples of the Venn diagram are as follows:

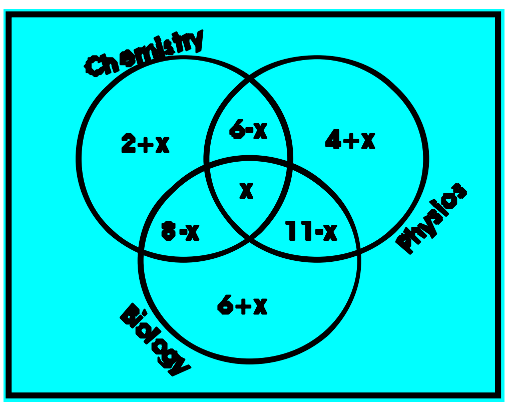




Example

In a class of 40 students, 25 offer Biology, 16 offer Chemistry, 21 offer Physics and each of the students must offer at least one of these three subjects. If 8 students offer Biology and Chemistry, 11 offer Biology and Physics and 6 offer Chemistry and Yoruba. Draw a Venn diagram to illustrate this information, using x to represent the number of students who offer all the three subjects. Calculate the value of x .

Solution; consider the Venn diagram below and using the cardinality of sets:



Let B denote the set of Biology, C the set of Chemistry and P the set of Physics.

$n(\varepsilon) = 40$

$n(B) = 25$

$n(C) = 16$

$n(P) = 21$

$n(\varepsilon) = 40$

Week 3

$n(B \cap C) = 8$

$n(B \cap P) = 11$

$n(C \cap P) = 6$

$n(B \cap C) = 8$

$n(B \cap C \cap P) = x$

$\therefore n(B \cap C \cap P') = 8 - x$

$\therefore n(B \cap C' \cap P) = 11 - x$

$\therefore n(B' \cap C \cap P) = 6 - x$

$\therefore n(B \cap C' \cap P') = 25 - (8 - x) - (11 - x) - x = 6 + x$

$\therefore n(B' \cap C \cap P') = 16 - (8 - x) - (6 - x) - x = 2 + x$

$\therefore n(B' \cap C' \cap P) = 21 - (6 - x) - (11 - x) - x = 4 + x$

Since all the parts in the Venn diagram must equal to the cardinality of the universal set we obtain the following equation.

$6 + x + 2 + x + 4 + x + 8 - x + 11 - x + 6 - x + x = 40$

$\therefore 37 + x = 40$

$\therefore x = 3$

1.11 Simple Identity of Sets

Equal sets

Two or more sets are equal if and only if they contain exactly the same elements regardless of the arrangements of the elements. For example sets $A = \{x, y, z\}$ and $B = \{z, y, x\}$ are equal. That is, $A = B$.

Week 3

Equivalent sets

Two or more sets are equivalent if and only if they contain exactly the same number of elements. Here we are only talking in terms of the number of elements but not the actual elements. For example sets $A = \{x, y, z\}$ and $B = \{1,2,3\}$ are equivalent. That is, $A \equiv B$.

Cardinal number of a set

The cardinal number of a set or simply the cardinality of a set is the number of elements in that particular set. The cardinal number of a set A is denoted by $n(A)$. For example, the cardinal number of the sets $A = \{x, y, z\}$ and $B = \{\ \}$ are respectively given as $n(A) = 3$ and $n(B) = 0$. The cardinality of a set is closely related to the probability theory as we shall see later.

Miscellaneous Examples in Set

1. If the universal set $U = \{1,2,3,4,5,6,7,8,9,10\}$, and the subsets $X = \{1,2,4,6,7,8,9\}$, $Y = \{1,2,3,4,6,7,9\}$ and $Z = \{2,3,4,7,9\}$, what is $X \cap Y \cap Z'$

Solution; here we need to find $Z' = \{1,5,6,8,10\}$.

$\therefore X \cap Y \cap Z' = \{1,6\}$

2. Given the universal set $U = \{\text{integers} \leq 20\}$, $P = \{\text{multiples of } 3\}$ and $Q = \{\text{multiples of } 4\}$, what are the elements of $P' \cap Q$

Week 3

Solution; for simplicity, we need to write these set in a set builder notation.

$U = \{1,2,3,4,...,20\}$

$P = \{3,6,9,12,15,18\}$

$Q = \{4,8,12,16,20\}$

$\therefore P' = \{1,2,4,5,7,8,10,11,13,14,16,17,19,20\}$

$\therefore P' \cap Q = \{4,8,16,20\}$

Week 3 Practical Activities

The Application of Sets in Medical Research

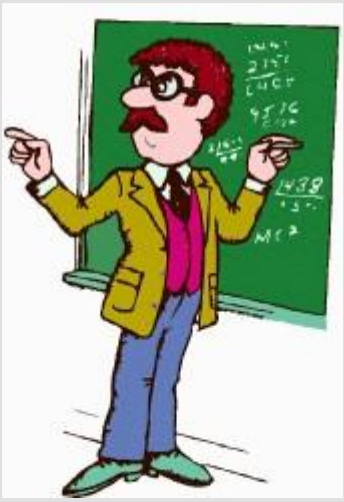
Students are required to visit a nearby hospital, clinic or health centre to investigate the prevalence of three common child-killer diseases. A sample of 200 children attending the clinic should be considered. The students should split themselves into three groups with each group enumerating the children infected with a particular disease. For instance, group 1 should find out, among the 200 children, how many are infected with measles. Group 2, should find out, among the 200 children, how many are infected with whooping cough and group 3 to enumerate how many are infected with diarrhoea. It is expected that the students should use their knowledge of set theory to find out the children infected with exactly two of the diseases. Also students should use their knowledge of set theory to investigate how many children are infected with all the three diseases.

The Application of Sets to Electioneering and Voting Coalition

Students are required to visit a nearby locality, town or village to investigate the popularity of three political parties in Nigeria. A sample of 300 adults with voters' registration cards should be considered. The students should split themselves into three groups with each group enumerating the people with a particular political affiliation. For instance, group 1 should find out, among the 300 people, how many are affiliated to party A. Group 2, should find out, among the 300 people, how many are affiliated to party B and group 3 should find out how many are affiliated to party C. It is expected that the students should use their knowledge of set operations to find out the people affiliated to exactly two of the political parties. Also students should use their knowledge of set operations to investigate how many people are affiliated to all the three political parties.

Week 3

WEEK 4



2.1 Mapping with Examples

Mapping is an assignment of elements between two or more sets through a well defined relationship. If for instance, to each element of set X, there is an assignment which links that element in X to an element in set Y, then this assignment is called mapping. Hence we write:

$f : X \rightarrow Y$

This statement is read X is mapped into Y. Then X is called the ***domain*** of *f*, and Y is the ***co-domain*** of *f*. If $x \in X$, then the element in Y which is assigned to *x* is called the image of *x* and it is denoted by $f(x)$. At times $f(x)$ is referred to as the value of *f* at *x*.

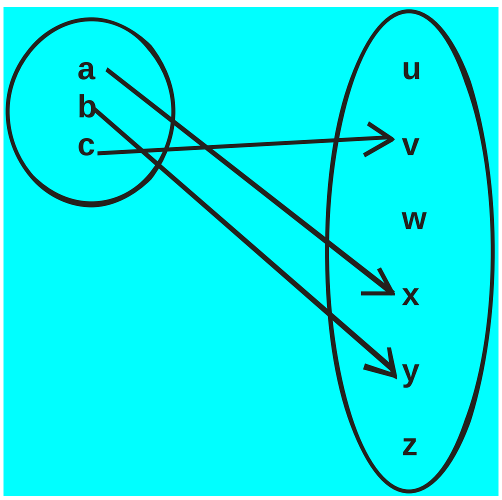
Range

The range of a mapping or a function is the set of images $f(X)$ written as:

$f(X) = \{f(x) / x \in X\}$. There are some mappings or functions in which the range and co-domain are equal and other mappings or functions in which they are not equal.

Example

Consider the mapping $f: X \rightarrow Y$, where $X = \{a, b, c\}$ and $Y = \{u, v, w, x, y, z\}$ defined by the following diagram:



This mapping is rightly a function because each element in X is assigned to some element in Y. Hence, a mapping could be function if and only if every element in the domain is assigned to a unique element in the co-domain.

2.2 Functions with Examples

A function $f: X \rightarrow Y$ is a rule or a relation which associates with each element $x \in X$ a unique element $y \in Y$ such that yfx . Usually, we denote the element $y \in Y$ for which yfx

by ‘ $f(x)$’. That is, $y = f(x)$. Again, the element *y* or $f(x)$ is called the image of *x* under *f*. The set X is called the domain of *f* and the set Y is the co-domain of *f*.

X: domain of *f* $x \xrightarrow{f} y = f(x)$ Y: co-domain of *f*

Basically, there are two points in the above definition of a function that require attention. First, the rule must be applicable to each element $x \in X$, there must be some $y \in Y$ which is f-related to *x*, i.e. $f : x \to y$. Second, this *y* must unique corresponding to the given *x*. in other words, for each $x \in X$, there is one and only one $y \in Y$ which is f-related to *x*.

Domain and Co-domain of a Function

From the definition of a function $f : X \to Y$ is read “*f* is a function of X into Y. The set X is called the domain of the function *f* and Y is called the co-domain of *f*. Furthermore, if $x \in X$ then the element in set Y which assigned to *x* is called the image of *x* and is denoted by $f(x)$ and reads “f of x”

Examples

Week 4

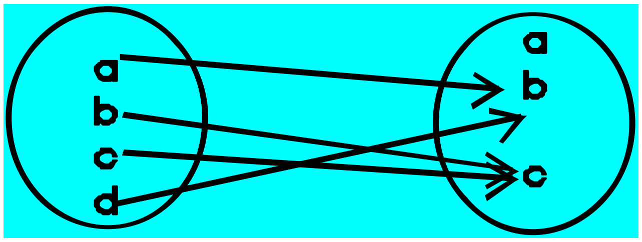
1. Let f assigned to each real number $\Re$ its square, i.e. for every real number x let $f(x) = x^2$. Then the domain and co-domain of f are both real numbers. i.e. $f:\Re \to \Re$. For example, the image of -3 is 9 and we write $f(-3) = 9$ or $f:-3 \to 9$.
2. Let f assigned to each country in the world its capital city. Here the domain of *f* is the set of countries in the world; the co-domain of f is the list of all capital cities in the world. i.e. $f(Nigeria) = Abuja$, which means that the image of Nigeria.

<u>Range of a Function</u>

Let $f: X \to Y$; we define the range of *f* to consist precisely of those elements in Y which appear as the image of at least one element in X. We denote the range of $f: X \to Y$ by $f(X)$.

Examples

1. Let the function $f:\Re \to \Re$ be defined by the formula $f(x) = x^2$. Then the range of *f* are the positive real numbers and zero.
2. Let $f: X \to Y$ be the function defined by $f(a) = b$, $f(b) = c$, $f(c) = c$, $f(d) = b$ where $X = \{a,b,c,d\}$ and $Y = \{a,b,c\}$. Then the range $f(X) = \{b,c\}$



Equal Functions

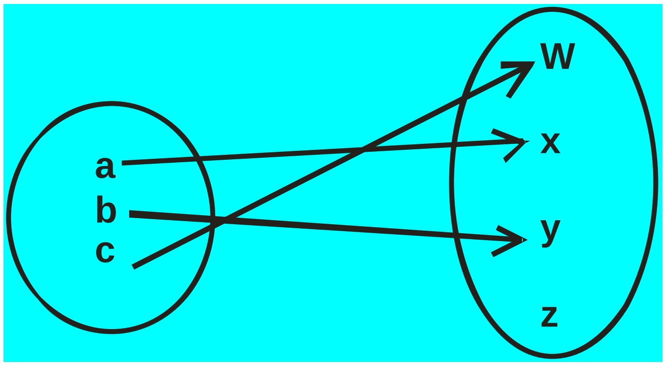
If *f* and *g* are functions defined on the same domain D and if $f(a) = g(a)$ for $a \in D$, then the functions *f* and *g* are equal and we write $f = g$

Examples

1. Let $f(x) = x^2$ where *x* is a real number. Let $g(x) = x^2$ where *x* is a complex number. Then the function *f* is not equal to *g* since they have different domains.
2. Let $\Re$ represent real number. Let $f : \Re \to \Re$ and $g : \Re \to \Re$. Suppose *f* is defined by $f(x) = x^2$ and *g* by $g(y) = y^2$. Then *f* and *g* are equal functions.

One-to-one Functions

Suppose *f* maps X into Y, then *f* is called a one-to-one function if different elements in Y are assigned to different elements in X. In other words, if no two different elements in X have the same image. More briefly, $f : X \to Y$ is one-to-one if $f(a) = f(a')$ implies $a = a'$ or equivalently, $f(a) \neq f(a')$ implies $a \neq a'$.



Examples

1. Let the function $f:\Re \rightarrow \Re$ be defined by the formula $f(x) = x^2$. Then f is not a one-one function since $f(2) = f(-2) = 4$.
2. Let f assigned to each country in the world its capital city; for example $f(Nigeria) = Abuja$ is an example of one-one function because no one country will have more than one capital city.

Week 4 Practical Activities

The Application of Functions to Probability Distributions

A lot of twelve television sets include two that are defective. If three of the sets are chosen at random for shipment to a hotel, how many defective sets can they expect?

Assuming that the possibilities are all equally likely, we find that the probability of x, the number of defective sets shipped to the hotel, is given by the function:

$$f(x) = \frac{\binom{2}{x}\binom{10}{3-x}}{\binom{12}{3}} \quad for \quad x = 0, 1, 2$$

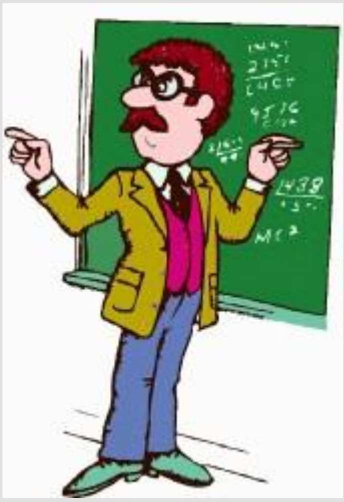
This can be expressed in a tabular as follows:

x	0	1	2
$f(x)$	$\frac{6}{11}$	$\frac{9}{22}$	$\frac{1}{22}$

Students are expected to express the following probability distribution function in a tabular form:

$$f(x) = \binom{5}{x}\left(\frac{1}{5}\right)^{x}\left(\frac{4}{5}\right)^{5-x} \quad for \quad x = 0, 1, 2, 3, 4, 5$$

WEEK 5

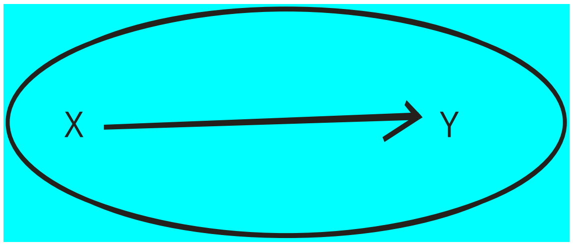


2.3 Difference between Mapping and Function

Basically, there are two points in the above definition of a function that require attention. First, the rule must be applicable to each element $x \in X$, there must be some $y \in Y$ which is f-related to *x*, i.e. $f : x \rightarrow y$. Second, this *y* must unique corresponding to the given *x*. In other words, for each $x \in X$, there is one and only one $y \in Y$ which is f-related to *x*. On the other hand, the second condition is not necessarily for mapping

2.4 Relations with Examples

A relation is a way of establishing a bond between the elements of two or more sets or between the elements in the same set. Suppose X and Y are sisters, we can use the diagram below to describe the bondage or relation between them.



If X is the sister of Y, then Y is also the sister of X. The double headed arrow is used in this case to show that the bondage between X and Y is reversible. Relation is particularly part of the open sentence of the type "is greater than", or "is sister of", or "is thrice as large as" and so forth.

Example

Consider the multiples of 3 on the set X, where $X = \{3,5,6,7,9,10\}$; in this example, 5, 7 and 10 are not multiples of 3 and they are not linked to 3. This example shows that 3 is related to 3, 6 and 9 and we write 3R3, 3R6, and 3R9. Since 3 is not related to 5, 7 and 10, we write $3\not R5,\ 3\not R7,\ and\ 3\not R10$.

A relation is a proposition which may be true or false, hence, that makes it different from a function. For example, let $X = \{Player\}$, $Y = \{Country\}$ and let the proposition $P(x, y)$ denote x plays football for y i.e. $P(x, y) = 'x\ plays\ football\ for\ y'$. Then the proposition $P(Okocha, Ghana)$ is false, whereas it is a relation.

Week 5

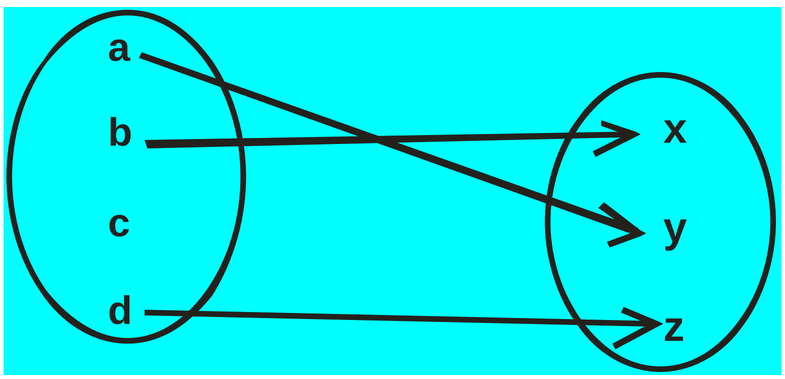
We call R a relation from X to Y and it is usually denoted by $R = \{X, Y, P(x, y)\}$.

Furthermore, if *P(x,y)* is true, we write *xRy* and reads *"x is related to y*". But is *P(x,y)* is false, we write $x\not R y$ and reads *"x is not related to y*".

Onto Functions

Let $f : X \to Y$, then the range $f(X)$ of the function *f* is a subset of Y i.e. $f(X) \subset Y$.

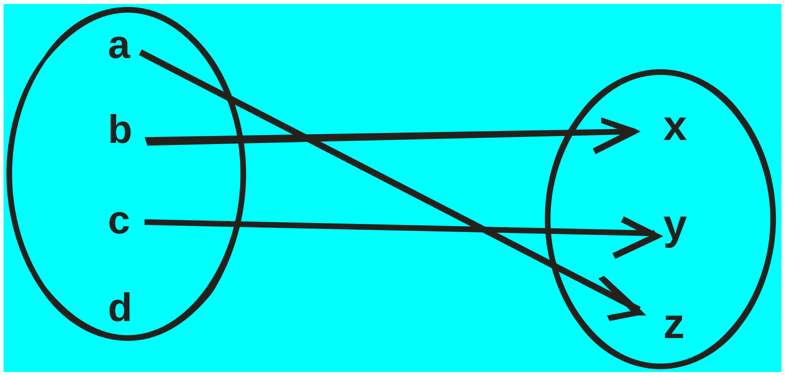
If $f(X) = Y$ i.e. if every member of Y appears as the image of at least one element of X, then we say *f* is a function of X onto Y. Or *f* maps X onto Y or *f* is an onto function. In other words, every element of Y is an image of some elements in X.



Examples

1. Let the function $f : \Re \to \Re$ be defined by the formula $f(x) = x^2$. Then *f* is not an onto function since the negative numbers do not appear in the range of *f*, i.e. no negative number is the square of real number.

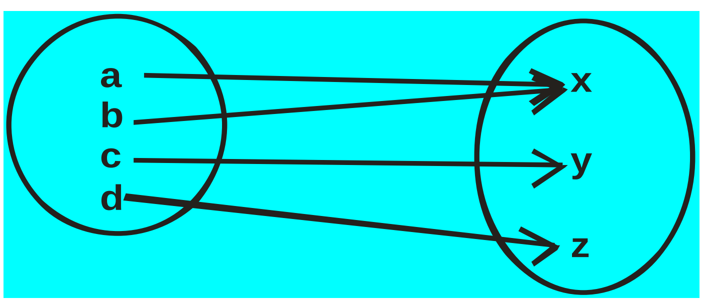
2. Let $X = \{a, b, c, d\}$ and $Y = \{x, y, z\}$; also let $f : X \to Y$ be defined by the following diagram;



Then $f(X) = \{x, y, z\} = Y$; i.e. the range of f equals the co-domain Y. Thus *f* maps X onto Y. i.e. *f* is an onto function or mapping.

Into Functions

Consider the function or mapping when every element of the set X has its image in the set Y and no member of X is left out without its image. This type of function or mapping is called mapping of X into Y. Here it is possible that more than one element of X are mapped into one element of Y.



Identity Function

Let X be any set; also let the function $f : X \to X$ be defined by the formula $f(x) = x$ i.e. let *f* assign to each element in X element itself. Then *f* is called the identity function. This is denoted by I or I_X. For example, any set could be an identity set.

Week 5

Constant Functions

A function $f : X \rightarrow Y$ is called a constant function if the same element of $y \in Y$ is assigned to every element in X. In other words, $f : X \rightarrow Y$ is a constant function if the range of *f* consists of only one single element.

Example

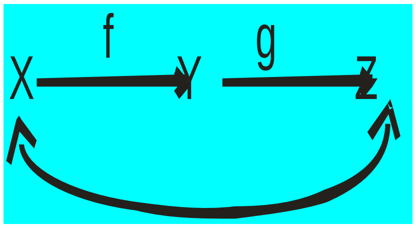
Let the function $f : \Re \rightarrow \Re$ be defined by the formula $f(x) = 5$; the f is a constant function since 5 is assigned to every element.

Product Function (Composite Function)

Suppose *f* maps X into Y and *g* maps Y into Z i.e. $X \xrightarrow{f} Y \xrightarrow{g} Z$. Let $x \in X$; then $f(x)$, the image of *x* is in Y the domain of *g*. We can similarly find the image of $f(x)$ under the function *g*, which is $g(f(x))$. Thus we have a rule which assigns to each element $x \in X$ a corresponding element $g(f(x)) \in Z$ i.e. we have a function of X into Z. This new function is called the product function and it is denoted by (gof).

In other words, if $f : X \rightarrow Y$ and $g : Y \rightarrow Z$, then we define a function $(gof): X \rightarrow Z$ by $(gof)(x) \equiv g(f(x))$. Thus the diagram:

$X \xrightarrow{f} Y \xrightarrow{g} Z$ simply becomes:



Example

Let the function *f* assign its square to each real number $\Re$ and let *g* assign the number plus 3 to each real number $\Re$. That is, let $f(x) = x^2$ and $g(x) = x + 3$. Find the product function (fog) and (gof) for (i) $4 \in \Re$ (ii) $x \in \Re$.

Solution

(i) $(fog)(4) \equiv f(g(4)) = f(7) = 49$

$(gof)(4) \equiv g(f(4)) = g(16) = 19$

(ii) $(fog)(x) \equiv f(g(x)) = f(x+3) = x^2 + 6x + 9$

$(gof)(x) \equiv g(f(x)) = g(x^2) = x^2 + 3$

Week 5

Exercise

1. Define function with two examples.

2. Define mapping with two examples.

3. Define relation with two examples.

4. Define and explain the domain and co-domain of a function.

5. Explain with example, the one-one function.

6. Explain with example, the onto function.

7. Explain with example, the constant function.

8. Explain with example, the equal function.

9. Explain with example, the identity function.

10. Explain with example, the image and range of a function.

Week 5 Practical Activities

Probability Application of Functions and Relations

Consider the following example for a lot of twelve television sets include two that are defective. If three of the sets are chosen at random for shipment to a hotel and assuming that the possibilities are all equally likely, we find that the probability of x, the number of defective sets shipped to the hotel, is given by the function:

$$f(x) = \frac{\binom{2}{x}\binom{10}{3-x}}{\binom{12}{3}} \quad \textit{for} \quad x = 0, 1, 2$$

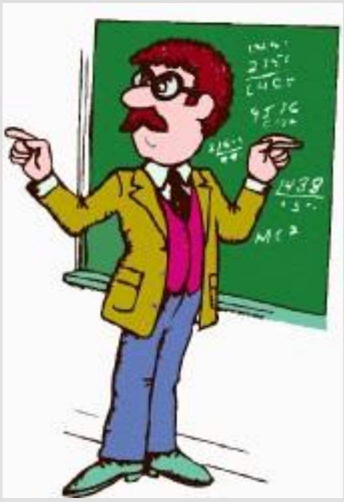
And expressed in the following table,

x	0	1	2
$f(x)$	$\frac{6}{11}$	$\frac{9}{22}$	$\frac{1}{22}$

Students are expected to express explain whether this probability distribution function is one-to-one or not. Also whether it is an onto function or not. This knowledge will later be applied to the application of random variables.

Week 5

WEEK 6



2.5 Difference between Function and Relation

Basically, a relation is a function in all ramifications; therefore the two words can be used interchangeably. Mathematically, the word function is preferred in terms of operations and mathematical computations. Here we shall consider more definitions of functions as follows:

Inverse Function

If we have a function $f : X \rightarrow Y$ then under the rule of mapping and function, we say that elements of the set X are mapped into set Y. However, the rule of function by which the set Y is mapped into the set X is called inverse function and is denoted by f^{-1}. In general, f^{-1} may not necessarily be a function. However, if f is both one-to-one and onto, then f^{-1} is a function form Y onto X and is called the inverse function. It is important to note that the domain of an inverse function is therefore the range of the original function. Similarly, the range of the inverse function represents the domain of the original function.

Again remember that a function *f* has an inverse if and only if it is both one-to-one and onto.

Example

Let $f : x \to x + 4$, find f^{-1}.

Solution

Here, we equate the function to a variable say *y* and we solve for *x* in terms of *y*. The procedures are as follows:

Let $x + 4 = y$

$\therefore x = y - 4$

Thus, using *x* as the starting value; that is replacing *y* by *x*, we simply get the inverse as follows:

$f^{-1} : x \to x - 4$

<u>Zeros of a Function</u>

The zeros of a function are the values of the function when it is equated to a zero.

Graphically, the zeros of a function are the points where the graph intersects the x-axis.

Week 6

Example

Given that $y = x^2 - 4$, find the zeros of the function *y*.

Solution

Here we set the entire function to be equal to zero and solve as follows:

$x^2 - 4 = 0$

$\therefore x = \pm 2$

Hence, -1 and 1 are the zeros of *y*, because $y = 0$ at $x = -2$ or $x = 2$

Singularities of a Function

The singularity of a function is the set of values of *x* for which *the* function is not defined. Graphically, singularities are the point on which the graph of the function is discontinuous. To find the singularity of a function, we simply equate its denominator to zero and solve for *x*.

Example

Given that $y = \frac{x^2 + 4}{x^2 - 5x + 6}$, find the singularities of the function y.

Solution

Here we set the denominator of the function to be equal to zero and solve as follows:

$x^2 - 5x + 6 = 0$

Solving the quadratic equation we have the singularities of the function y at

$x = 2 \ or \ x = 3$.

<u>Some Common Classes of a Function</u>

<u>Linear Function</u>

$f(x) = ax + b$, where a, b are real numbers. The graph of a linear function is always a straight line.

<u>Quadratic Function</u>

$f(x) = ax^2 + bx + c$, where a, b, c are real numbers. The graph of a quadratic function is always a parabola.

<u>Cubic Function</u>

$f(x) = ax^3 + bx^2 + cx + d$, where a, b, c, d are real numbers. The graph of a cubic function is two parabolas joined together.

Polynomial Function

$f(x) = ax^n + bx^{n-1} + ... + cx + d$, where a, b,… c, d are real numbers; is a polynomial of degree *n*. Thus a linear function is a polynomial function of degree one, a quadratic function is a polynomial function of degree two, a cubic function is a polynomial function of degree three, and so forth.

Exponential Function

$f(x) = a^x$; where a, is a real number.

Power or Geometric Function

$f(x) = x^n$

Logarithmic Function

$f(x) = \log_a^x$

Exercise

1. Two functions $f(x) = 2x$ and $g(x) = x + 3$ define in similar manner $(fog)(x)$ and $(gof)(x)$

2. Given that $f : x \to 3 - 2x$, find the inverse function f^{-1}

3. Find the zeros of the function $f(x) = x^2 + 9x + 20$

Week 6

4. Find the singularities of the function $f(x) = \dfrac{x^2 + 8x - 24}{2x^2 - 4x}$

5. The function $g(x) = \dfrac{1}{1+x}$, given that $g(x) = g(g(x))$, show that

$x^2 + x - 1 = 0$

6. Find the inverse of the function $f(x) = \dfrac{2}{1-x}$

7. If $f(x) = x + 2$ and $g(x) = x^2 + 1$, write down the functions: $f(x) + g(x)$ and

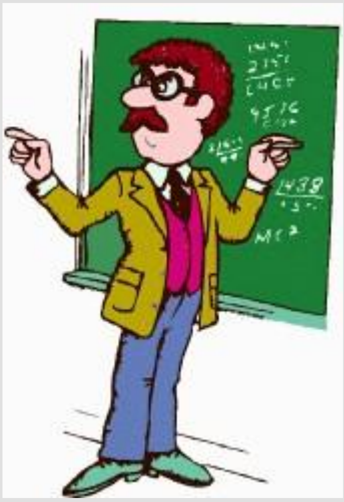
$f(x).g(x)$

Week 6 Practical Activities

The Statistical Application of Functions

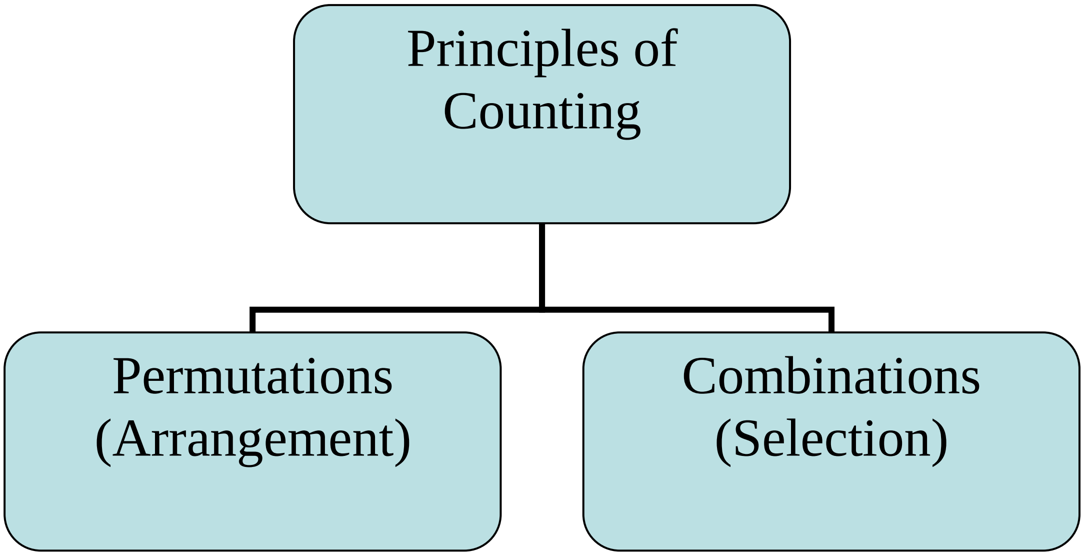
Students are required to visit the Federal Bureau of Statistics to obtain any probability distribution function for any social or economic variable. They are expected to use the function in generating the appropriate probability distribution tables.

WEEK 7



Permutations and Combinations

Permutations and combinations are basic prerequisite for understanding applied probability. Here we shall learn a method of approach to certain problems involving arrangements and selections in the course of the work, a notation is introduced, and formulae are used for both arrangements and selections. Moreover, we shall develop some techniques for determining without direct enumeration the number of possible outcomes of a particular experiment or the number of elements in a particular set. Such techniques are sometimes referred to as combinatorial analysis.



Fundamental Principles of Counting

If some procedure can be performed in n_1 different ways, and if, following this procedure, a second procedure can be performed in n_2 different ways, and if, following this second procedure, a third procedure can be performed in n_3 different ways, and so forth, then the number of ways the procedures can be performed in the order indicated is the product $n_1 \times n_2 \times n_3 \times ...$

Example

Suppose a number plate contains two distinct letters followed by three digits with the first digit not zero. How many different number plates can be printed?

Solution

The first letter can be printed in 26 different ways, the second letter in 25 different ways (since the letter printed first cannot be chosen for the second letter), the first digit in 9 ways and each of the other two digits in 10 ways. Hence we have:

$$26 \times 25 \times 9 \times 10 \times 10 = 585{,}000$$

Which implies the 585,000 different plates can be printed.

Factorial Notation

The product of the positive integers from 1 to 9 inclusive occurs very often in mathematics and hence is denoted by the special symbol n! (Read “n factorial”) and mathematically given as follows:

$$n! = n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1$$

It is also convenient to define $0! = 1$

Examples

1. $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$
2. $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

3.1 Permutations

An arrangement of a set of *n* objects in a given order is called a permutation of the object (taken all at a time). Arrangement of any $r \leq n$ of these objects in an order is called r-permutation or a permutation of *n* objects taken *r* at a time.

Example

Consider the set of letter a, b, c and d; Then we have the following:

(i) *dbca*, *dcba*, *acdb* are permutations at the 4 letters (taken all at a time).

(ii) *bad*, *adb*, *abd* and *bca* are permutations of the 4 letters taken 3 at a time.

(iii) *ad*, *cb*, and *bd* are permutations of the 4 letters taken 2 at a time.

The number o permutations of *n* objects taken *r* at a time will be denoted by ^nP_r. Before we derive the general formula for ^nP_r, we consider a special case.

Example

Find the number of permutations of 6 digits, say *a,b,c,d,e,f* taken three at a time. In other words, find the number of "three letter words" with distinct letters that can be formed from the above six letters.

Week 7

Let the general three letter word be represented by three boxes:-

Now the first letter can be chosen in 6 different ways; following this, the second letter can be chosen in 5 different ways; following this, the last letter can be chosen in 4 different ways. Write each number in its appropriate box as follows:

6	5	4

Thus by the fundamental principle of counting, there are $6 \cdot 5 \cdot 4 = 120$ possible three letter words without repetitions from the six letters, or there are 120 permutations of 6 objects taken 3 at a time. That is, ${}^{6}P_{3} = 120$.

The derivation of the formula for ${}^{n}P_{r}$ follows the procedure in the preceding example. The first element in an *r*-permutation of *n*-objects can be chosen in *n* different ways; following this, the second element in the permutation can be chosen in *n-1* ways, and following this, the third element in the permutation can be chosen in *n-2* ways. Counting in this manner, we have that the *rth* (last) element in the *r*-permutation can be chosen in $n-(r-1)=(n-r+1)$ ways. Thus:

Theorem

$${}^{n}P_{r} = n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$$

The second part of the formula follows from the fact that:

$$n(n-1)(n-2)\ldots(n-r+1) = \frac{n(n-1)(n-2)\ldots(n-r+1)(n-r)!}{(n-r)!} = \frac{n!}{(n-r)!}$$

In the special case that $r = n$, we have the following formula:

$^{n}P_{n} = n(n-1)(n-2)...3 \cdot 2 \cdot 1 = n!$

Namely,

Corollary:

There are n! Permutations of n objects (taken all at a time).

there are n! Permutations of n objects (taken all at a time).

Example

How many permutations are there of 3 objects, say a, b and c? By the above corollary, there are $3! = 3 \cdot 2 \cdot 1 = 6$ such permutations. There are: abc, acb, bac, bca, cab, cba.

3.2 Combinations

This is particularly used in some probability distributions like the binomial and hypergeometric. Suppose we have a collection of n objects. A combination of these n objects taken r at a time, or an r-combination, is any subset of r elements. In other words, an r-combination is any selection of r of the n objects where order does not count.

Example

Week 7

The combinations of the letters a,b,c,d, taken 3 at a time are {a,b,c,}, {a,b,d,}, {a,c,d}, {b,c,d} or simply abc, adb, acd, bcd

Observe that the following combinations are equal:
abc, acb, bac, bca, cab, cba

That is, each denotes the same set {a,b,c}

The number of combinations of *n* objects taken *r* at a time will be denoted by $^{n}C_{r}$.

Before we give the general formular for $^{n}C_{r}$, we consider a special case.

Example

We determine the number of combinations of the four letters a,b,c,d taken 3 at a time. Note that each combination consisting of three letters determines $3! = 3 \cdot 2 \cdot 1 = 6$ permutations of the letters in the combination.

Combinations	**Permutations**
abc	abc, acb, bac, bca, cab, cba
abd	abd, adb, bad, bda, dab, dba
acd	acd, adc, cad,cda, dac, dca
bcd	bcd, bdc, cbd, cdb, dbc, dcb

Thus the number of combinations multiplied by $3!$ equals the number of permutations:

$^4C_3 \cdot 3! = {}^4P_3$

Or $^4C_3 = \dfrac{^4P_3}{3!}$

Now $^4P_3 = 4 \cdot 3 \cdot 2 = 24$

Hence, $^4C_3 = 4$ as noted above

Since each combination of *n* objects taken *r* at a time determines $r!$ permutations of the objects, we can conclude that:

$^nP_r = r!\,{}^nC_r$

Theorem

$$^nC_r = \frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

Note that

$^nC_r = \dfrac{n!}{r!(n-r)!}$ can also be written as:

$\dbinom{n}{r}$; therefore, we have

$$^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

We shall use ^nC_r and $\dbinom{n}{r}$ interchangeably

Example

Week 7

How many committees of 3 can be formed from 8 people?

Solution

Each committee is essentially a combination of the 8 people taken 3 at a time. Thus:

$$^{8}C_{3} = \binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$$

different committees can be formed.

Note

Combination is actually the number of ways of selecting *r* objects from *n* objects without regards to order. In other words, combination is actually a partition with 2 cells, one cell containing the *r* objects selected and the other cell containing the remaining *(n-r)* objects.

Example

1) From 4 men and 3 women, find the number of committees of 3 that can be formed with 2 men and 1 woman.

Solution

The number of ways of selecting 2 men from 4 men is:

$${}^{4}C_{2} = \frac{4!}{2!(4-2)!} = 2$$

and the number of ways of selecting 1 woman from 3 women is:

$${}^{3}C_{1} = \frac{3!}{2!(3-2)!} = 3$$

hence, the number of committees that can be formed with 2 men and 1 woman =

$6 \times 3 = 18$

Example

Week 7

2) A mixed hockey team containing 5 men and 6 women is to be chosen from 7 men and 9 women. In how many ways can this be done?

Solution

Five men can be selected from 7 men in ${}^{7}C_{5}$ ways, and 6 women can be selected from 9 women in ${}^{9}C_{6}$ ways. Now for each of the ${}^{7}C_{5}$ ways of selecting the men, there are ${}^{9}C_{6}$ ways of selecting the women, therefore there are ${}^{7}C_{5}\times{}^{9}C_{6}$ ways of selecting the team. Thus;

$${}^{7}C_{5}\times{}^{9}C_{6} = \frac{7!}{2!5!}\cdot\frac{9!}{3!5!} = 21\times 84 = 1764$$

Therefore, the team can be chosen in 1764 ways.

3.3 Applications of Permutation and Combinations

Ordered Partitions

Suppose an urn *A* contains seven marbles numbered 1 through 7. We compute the number of ways we can draw, first, 2 marbles from the urn, the 3 marbles from the urn, and lastly 2 marbles from the urn. In other words, we want to compute the number of ordered partitions.

$$(A_1, A_2, A_3)$$

of the set of 7 marbles into cells A_1, containing 2 marbles, A_2 containing 3 marble and A_3 containing 2 marbles. We call these ordered partitions since we distinguish between.

$(\{1,2\},\{3,4,5\},\{6,7\})$ and $(\{6,7\},\{3,4,5\},\{1,2\})$

each of which yields the same partition of *A*

Since we begin with 7 marbles in the urn, there are $\binom{7}{2}$ ways if drawing the first 2 marbles, i.e., of determining the first cell A_1; following this, there are 5 marbles left in the urn and so there are $\binom{5}{3}$ ways of drawing the 3 marbles i.e. of determining A_2; finally, there are 2 marbles left in the urn and so there are $\binom{2}{2}$ ways of determining the last cell A_3. Thus there are:

$$\binom{7}{2}\binom{5}{3}\binom{2}{2} = \frac{7\cdot 6}{1\cdot 2}\cdot\frac{5\cdot 4\cdot 3}{1\cdot 2\cdot 3}\cdot\frac{1\cdot 2}{1\cdot 2} = 210$$

different ordered partitions of *A* into cells A_1 containing 2 marbles, A_2 containing 3 members, and A_3 containing 2 marbles.

Now observe that

$$\binom{7}{2}\binom{5}{3}\binom{2}{2} = \frac{7!}{2!5!}\cdot\frac{5!}{3!2!}\cdot\frac{2!}{2!0!} = \frac{7!}{2!3!2!}$$

Since each numerator after the first is cancelled by the second term in the denominator of the previous factor. In a similar manner as the permutations with repetitions we have:

Theorem

Let *A* contains *n* elements and let $n_1, n_2, ..., n_r$ be positive integers with $n_1 + n_2 +,...,+ n_r = n$. Then there consists:

$$\frac{n!}{n_1!n_2!n_3!...n_r!}$$

different ordered partitions of A of the form $(A_1, A_2,..., A_r)$ where A_1 contains n_1 elements, A_2 contains n_2 elements, … A_r contains n_r elements.

Example

In how many ways can 9 toys be shared among 4 children if the youngest child is to receive 3 toys and each of the other children 2 toys?

Solution

We wish to find the numbers of ordered partitions of the 9 toys into 4 cells containing 3, 2, 2, and 2 toys respectively. By the above theorem, there are

$$\frac{9!}{3!2!2!2!} = 7560$$

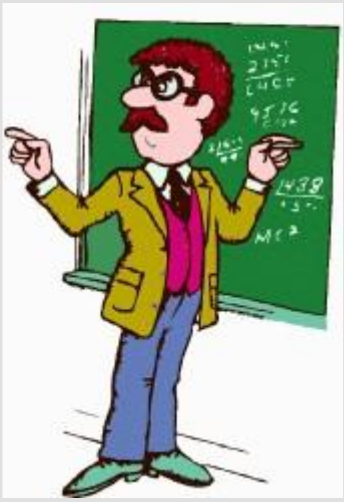
Such ordered partitions.

Week 7 Practical Activities

The Application of Permutations and Combinations

Students are required to visit the Federal Road Safety Corp (FSRC) and obtain relevant information on the number of digits and letters used for a number plate. On the basis of this information, students are required to use their knowledge on permutations and combinations to determine the possible number of plates expected to be produced.

WEEK 8



3.4 Experiments with Permutation and Combinations

Permutations with Repetitions

Frequently we want to know the number of permutations of objects some of which are alike, as illustrated below:

The general formula is as follows:

Theorem

The number of permutations of *n* objects of which n_1 are alike, n_2 are alike,…, n_r are alike is:

$$\frac{n!}{n_1!n_2!...n_r!}$$

We indicate the proof of the above theorem by a particular example. Suppose we want to form all possible 5 letter words using the letters from the word *DADDY*. Now there are $5! = 120$ permutations of the objects D_1, A, D_2, D_3, Y where the three *D's* are distinguished, observe that the following six permutations.

$D_1\ D_2\ D_3\ AY$	$D_2\ D_1\ D_3\ AY$	$D_3\ D_1\ D_2\ AY$
$D_1\ D_3\ D_2\ AY$	$D_2, D_3, D_1\ AY$	$D_3\ D_2\ D_1\ AY$

Produce the same word when the subscripts are removed. The 6 comes from the fact that there are $3! = 3 \cdot 2 \cdot 1 = 6$ different ways if placing the three *D's* in the first three positions in the permutation. This is true for each of the other possible positions in which the *D's* appear. Accordingly there are:

$$\frac{5!}{3!} = \frac{120}{6} = 20$$

That is, 120 different 5 letters words that can be formed using the letters from the word *DADDY*.

Example

How many different signals, each consisting of 8 flags hung in a vertical line, can be formed from a set of 4 indistinguishable red flags, 3 indistinguishable white flags and a blue flag?

Solution

We seek the number of permutations of 8 objects of which 4 are alike (the red flags) and 3 are alike (the white flag). By the above theorem, there are:

$$\frac{8!}{4!3!} = \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{4\cdot 3\cdot 2\cdot 1\cdot 3\cdot 2\cdot 1} = 280,$$

That is, there are 280 different signals.

Ordered Samples

Many problems in combinational analysis and, in particular, probability are concerned with choosing a ball from an urn containing *n* balls (or a card from a deck, or a person from a population). When we choose one ball after another from the urn, say *r* times, we call the choice an ordered sample of size *r*. Here we shall consider two cases.

i) **Sampling with Replacement**

Here the ball is replaced in the urn before the next ball is chosen. Now since there are *n* different ways to choose each ball, there are by the fundamental principle of counting we have:

$\underbrace{n\cdot n\cdot n\cdot \ldots\cdot n}_{r\ times} = n^r$ different ordered samples with replacement of size r.

ii) **Sampling without Replacement:**

Here the ball is not replaced in the urn before the next ball is chosen. This there are no repetitions in the ordered sample. In other words, an ordered sample of size *r* without replacement is simply an *r*-permutation of the objects in the urn. Thus there are:

$$^{n}P_{r} = n(n-1)(n-2)...(n-r+1) = \frac{n!}{(n-r)!}$$

Different ordered samples or size r without replacement from the population of n objects.

Example

In how many ways can one choose three cards in succession from a deck of 52 cards.

(i) With replacement?

(ii) Without replacement?

Solution

If each care is replaced in the deck before the next card is chosen, then each card can be chosen in 52 different ways. Hence there are:

$52.52.52 = 52^3 = 140,608$

different ordered samples of size 3 with replacement; on the other hand if there is no replacement, then the first card can be chosen in 52 different ways, the second care is 51 different ways, and the third and last card in 50 different ways. Thus there are

$52.51.50 = 132, 600$

That is there are 132,600 different ordered samples of size 3 without replacement.

Permutation in a Circle

The number of permutations of *n* distinct objects arranged in a circle is $(n-1)!$

Example

In how many ways can 7 people be seated at a round table if:

(a) They can sit anywhere?

(b) Two particular people must not sit next to each other?

Solution

a) Let one of them be seated anywhere, then the remaining 6 people can be seated in $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$ ways; which is the total numbers of ways of arraying the *7* people in a circle i.e., $(n-1)!$ ways.

b) Consider the two particular people as one person. Then there are 6 people altogether and they can be arranged in $5!$ ways. But the two people considered as one person can be arranged between themselves in $2!$ Ways. Thus the number of ways of arranging 6 people at a round table with two people sitting together is $5!2! = 240$. Then the total number of ways in which seven people can be seated at a round table so that the two people do not sit together is: *720-240 = 480* ways.

Week 8

Combination with Ordered Partitions

Suppose an urn A contains seven marbles numbered 1 through 7. We compute the number of ways we can draw, first, 2 marbles from the urn, the 3 marbles from the urn, and lastly 2 marbles from the urn. In other words, we want to compute the number of ordered partitions.

(A_1, A_2, A_3)

of the set of 7 marbles into cells A_1, containing 2 marbles, A_2 containing 3 marble and A_3 containing 2 marbles. We call these ordered partitions since we distinguish between.

$(\{1,2\},\{3,4,5\},\{6,7\})$ and $(\{6,7\},\{3,4,5\},\{1,2\})$

each of which yields the same partition of A

Since we begin with 7 marbles in the urn, there are $\binom{7}{2}$ ways if drawing the first 2 marbles, i.e., of determining the first cell A_1; following this, there are 5 marbles left in the urn and so there are $\binom{5}{3}$ ways of drawing the 3 marbles i.e. of determining A_2; finally, there are 2 marbles left in the urn and so there are $\binom{2}{2}$ ways of determining the last cell A_3. Thus there are:

$$\binom{7}{2}\binom{5}{3}\binom{2}{2} = \frac{7\cdot 6}{1\cdot 2}\cdot\frac{5\cdot 4\cdot 3}{1\cdot 2\cdot 3}\cdot\frac{1\cdot 2}{1\cdot 2} = 210$$

different ordered partitions of A into cells A_1 containing 2 marbles, A_2 containing 3 members, and A_3 containing 2 marbles.

Now observe that

$$\binom{7}{2}\binom{5}{3}\binom{2}{2} = \frac{7!}{2!5!}\cdot\frac{5!}{3!2!}\cdot\frac{2!}{2!0!} = \frac{7!}{2!3!2!}$$

Since each numerator after the first is cancelled by the second term in the denominator of the previous factor. In a similar manner as the permutations with repetitions we have:

Theorem

Let A contains n elements and let $n_1, n_2, ..., n_r$ be positive integers with

$n_1 + n_2 +,...,+ n_r = n$. Then there consists:

$$\frac{n!}{n_1!n_2!n_3!...n_r!}$$

different ordered partitions of A of the form $(A_1, A_2, ..., A_r)$ where A_1 contains n_1 elements, A_2 contains n_2 elements, … A_r contains n_r elements.

Example

In how many ways can 9 toys be shared among 4 children if the youngest child is to receive 3 toys and each of the other children 2 toys?

Solution

We wish to find the numbers of ordered partitions of the 9 toys into 4 cells containing 3, 2, 2, and 2 toys respectively. By the above theorem, there are

$$\frac{9!}{3!2!2!2!} = 7560$$

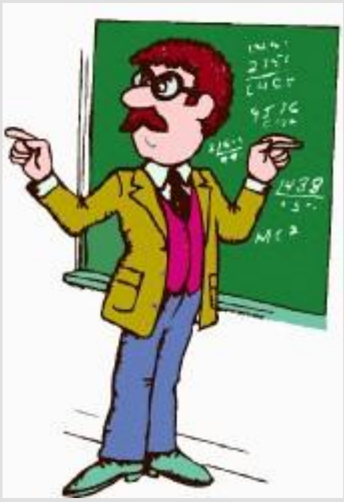
Such ordered partitions.

Week 8 Practical Activities

The Application of Permutations and Combinations to Business

How many 12 digits mobile telecom PIN numbers can be produced if a digit can be used more than once? Students are expected to visit any of the mobile telecom companies and obtain relevant information as the number of subscribers and use permutations and combinations to obtain the number of cards produced.

WEEK 9



4.1 Statistical Experiment

A statistical experiment is any process which results in the collection of data. This meaning for the word experiment is somewhat different from that usually given to it in other scientific disciplines. Probability is the study of random or non-deterministic ***experiments***. If a die is tossed in the air, then it is certain that the die will come down, but it is not certain that, say, a 6 will appear. However, suppose we repeat this experiment of tossing a die, let s be the number of successes, i.e. the number of times a 6 appears, and let n be the number of tosses. Then it has been empirically observed that the ratio $f = \frac{s}{n}$ called a relative frequency, become stable in the long run i.e. approached a limit. This stability is the basis of probability theory.

Definition of Probability of an Event

The probability of any event A is the sum of the weights of all sample points in A. Therefore:

$$0 \le P(A) \le 1, \quad P(\Phi) = 0, \quad P(S) = 1$$

In probability theory, we define a mathematical model of the above phenomenon by assigning "probabilities" (or: the limit values of the relative frequencies) to the "events" connected with an experiment. Naturally, the reliability of our mathematical model for a given experiments depends upon the closeness of the assigned probabilities to the actual relative frequency. This then gives rise to problems of testing and reliability which form the subject matter of statistics.

Historically, probability theory began with a study of games of chance, such as die and card. The probability *P* of an event *A* was defined as follows:
If *A* can occur in *S* ways out of a total on *n* equally likely ways, then:

$$P = P(A) = \frac{s}{n}$$

For example, in tossing a die an even number can occur in 3 ways of 6 "equally likely" ways, hence $P = \frac{3}{6} = \frac{1}{2}$. This classical definition of probability is essentially circular since the idea of "equally likely" is the same as that of "with equal probability" which has not been defined. The modern treatment of probability theory is purely axiomatic. This means that the probabilities of our events can be perfectly arbitrary, except that they must satisfy certain axioms listed below. The classical theory will correspond to the special case of so-called equiprobable spaces.

Example

When 1084 adults were surveyed in Nigeria, 813 indicated support for a ban on gay marriage. Use these survey results to estimate the probability that randomly selected adults would support such a ban.

Solution

$$P = P(\text{support a ban}) = \frac{813}{1084} = 0.75$$

4.2 Sample Space and Sample Point

The set S of all possible outcomes of some given experiment is called the ***sample space***. A particular outcome i.e. an element in S is called a ***sample point*** or ***sample***.

4.3 Construction of sample space

It is pertinent to learn the construction of sample space for compound events.

Examples

(1) A pair of dice is rolled once, show the sample space S and the event A of getting a sum of 11.

S	**1**	**2**	**3**	**4**	**5**	**6**
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

Here, the sample space *S* has 36 sample points and the event $A = \{(5,6), (6,5)\}$

1) Consider an experiment where we are interested in the sum of the outcomes of a pair of dice thrown once. This is a situation where we are more interested in the sum of the outcome rather than their physical combinations.

+	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Note

In probability theory rolling a pair of dice once is equivalent to rolling a single die twice in terms of outcomes and sample space.

3) Consider an experiment where a single die and a coin a tossed simultaneously. Show the sample space and an event of obtaining a head and an event number.

Solution

Week 9

		Die					
		1	**2**	**3**	**4**	**5**	**6**
Coin	**H**	1H	2H	3H	4H	5H	6H
	T	1T	2T	3T	4T	5T	6T

$A = \{2H, 4H, 6H\}$

4) Consider an experiment where a coin is tossed three times. Show the sample space *S* and an event *C* of obtaining exactly two heads.

Solution

First, the sample space of two coin is as follows:

		First coins	
		H	**T**
Second coin	**H**	HH	HT
	T	TH	TT

Second, the sample space of three coins is as follows

	Two coins			
	HH	**HT**	**TH**	**TT**

One coin					
	H	HHH	HHT	HTH	HTT
	T	THH	THT	TTH	TTT

$B = \{HHT, HTH, THH\}$

Week 9 Practical Activities

Practical Applications of Experimental Probability

1. If the birth of a male or a female child is assumed to be equally likely, considering only families with three children, students should calculate the probabilities for the following table. Where B denotes boy and G denotes girl. Students are also required to visit 50 households with three children and calculate these probabilities by relative frequency approach and compare the results.

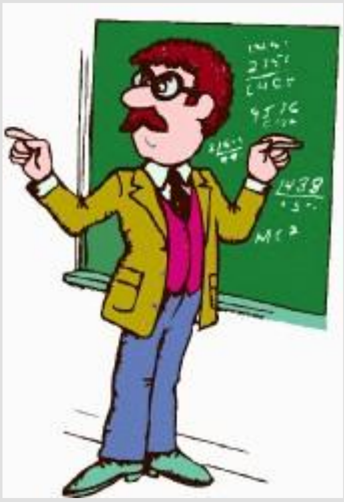
Birth order	**BBB**	**BBG**	**BGB**	**BGG**	**GBB**	**GBG**	**GGB**	**GGG**
Probability	…	…	…	…	…	…	…	…

2. In a recent year, Nigeria experienced 68,593 vehicle accidents with 26,201 of them involving reportable property damage. Use these results to estimate the probability that a random Nigerian accident results in reportable property damage.
3. Among 80 randomly selected blood donors, 36 were classified as group O. What is the approximate probability that a person will have group O blood?

4. If a person is randomly selected, find the probability that his or her birthday is May 1, which is National Workers Day in Nigeria. Ignore leap year.

Week 9

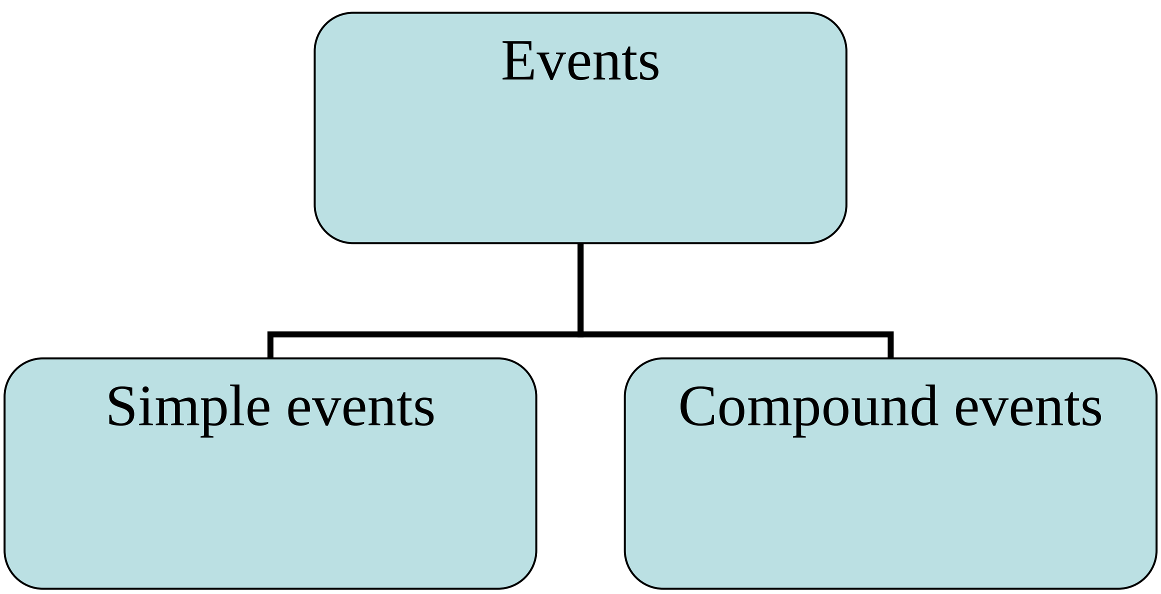
WEEK 10



4.4 Events

An event *A* is a set of outcomes or, in other words, a subset of the sample space *S*. The event {a} consisting of a single sample $a \in S$ is called an elementary event. The empty set ϕ and *S* itself are events; ϕ is sometimes called the impossible event, and *S* the certain or sure event.

4.5 Simple and Compound Events



Simple event

If an event is a set containing only one element of the sample space, then it is called a simple event.

Example 1

Suppose we have balls in a bag labeled B_1, W_2, R_3, R_4 and R_5. Then the event of drawing a blue ball from this bag is the subset $A = \{B_1\}$ of the sample space $S = \{B_1, W_2, R_3, R_4, R_5\}$ Therefore, A is a simple event.

Example 2

In a recent National election, there were 25,569,000 citizens in the 18-24 age brackets. Of these, 9,230,000 actually voted. Find the empirical probability that a person randomly selected from this group did vote in the national election.

Solution

$$P = P(voting) = \frac{9230000}{25569000} = 0.361$$

Compound event

A compound even is the one that can be expressed as the union of simple events.

Let us consider the example given above under simple event, then the event *B* of drawing a red ball is a compound event since $B = \{R_3 \cup R_4 \cup R_5\} = \{R_3, R_4, R_5\}$. The union of simple events produces a compound event that is still a subset of the sample space.

Example 1

An experiment involves tossing 3 fair coins, and recording the faces that comes up.

a. List the elements of the sample space *S*
b. List the elements of s corresponding to the event *A* that the number of heads is greater than one.
c. List the elements of *S* corresponding to event *B* that a tail occurs in either coins.

Solution

Week 10

(a) *S = {HHH, HHT, HTH, HTT, THH, THT,TTH, TTT}*

(b) *A = {HHH. HHT, HTH, THH}*

(c) *B = { HHT, HTH, HTT, THT, TTH, THH, TTT}*

Example 2

Experiment: Toss a die and observe the number that appears on top. Then the sample space consists of the six possible numbers:

$S = \{1,2,3,4,5,6\}$

Let *A* be the event that an even number occurs, *B* that an odd number occurs and *C* that prime number occurs:

$A = \{2,4,6\}$,

$B = \{1,3,5\}$,

$C = \{2,3,5\}$

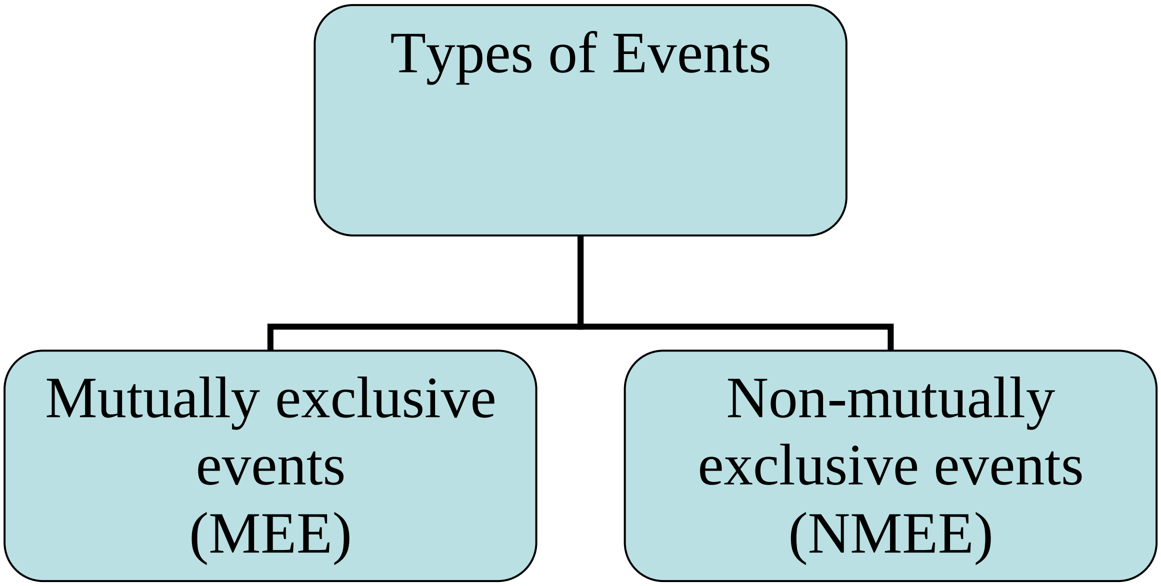
Then:

$A \cup C = \{2,3,4,5,6\}$ is the event that an even or a prime number occurs

$B \cap C = \{3,5\}$ is the event that an odd and a prime number occurs

$C' = \{1,4,6\}$ is the event that a prime number does not occur.

Note that *A* and *B* are mutually exclusive": $A \cap B = \phi$; in other words, an even number and an odd number cannot occur simultaneously.



4.6 Mutually Exclusive Events (MEE)

Two event E_1 *and* E_2 are mutually exclusive if and only if $E_1 \cap E_2 = \phi$. In other words, two events are mutually exclusive if they have no points in common.

Example

Suppose we throw a balanced die. Let E_1 be the event that an even number turns up and let E_2 be the event that an odd number shows. Then

$S = \{1,2,3,4,5,6\}$

$E_1 = \{2,4,6\}$

$E_2 = \{1,3,5\}$

and their intersection i.e. $E_1 \cap E_2 = \phi$ since they have no point in common. Hence, E_1 *and* E_2 are mutually exclusive events.

If E_1 *and* E_2 are mutually exclusive events, then:

$$P(E_1 \text{ or } E_2) = P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

Generalizing this case; if $E_1, E_2, ..., E_n$ are mutually exclusive events then:

$$P(E_1 \cup E_2 \cup E_3 \cup .... \cup E_n) = P(E_1) + P(E_2) + P(E_3) + ... + P(E_n)$$

Note

If $E_1, E_2, ..., E_n$ is a partition of a sample space *S*, then;

$$P(E_1 \cup E_2 \cup E_3 \cup .... \cup E_n) = P(E_1) + P(E_2) + P(E_3) + ... + P(E_n) = P(S) = 1$$

Example 1

A bag contains 4 red, 5 yellow and 6 black identical balls. A ball is selected at random, what is the probability that the ball is yellow or black?

Solution

The events of a yellow or black ball are mutually exclusive because a ball can only belong either of the sets of yellow or black balls but not both.

$P(Yellow\ ball) = P(Y) = \frac{5}{15} = \frac{1}{3}$

$P(Black\ ball) = P(B) = \frac{6}{15} = \frac{2}{5}$

$\therefore P(Yellow\ or\ black\ ball) = P(Y) + P(B) = \frac{1}{3} + \frac{2}{5} = \frac{11}{15}$

Example 2

A computer is used to generate random telephone numbers. Of the numbers generated and in series, 56 are unlisted, and 144 are listed in the telephone directory. If one of these telephone numbers is randomly selected, what is the probability that it is unlisted?

Solution

$$P = P(unlisted\ \ number) = \frac{56}{200} = 0.28$$

Week 10

Exercises

1) A die is loaded in such a way that an even number is twice as likely to occur as an odd number. If *A* is the event that a number less than 4 occurs on a single toss of the die, find *P(A)*.

2) If a card is drawn from an ordinary deck of 52 cards, find the probability that it is a heart.

Week 10 Practical Activities

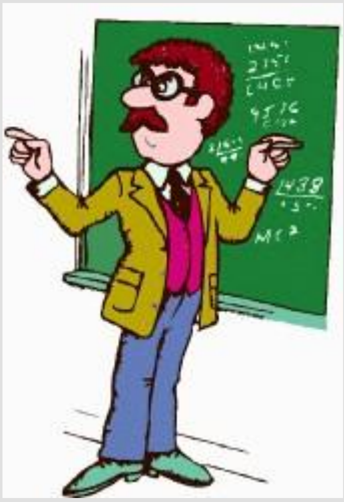
Application of Probability

1. In a random sample of 1000 men, 85 were fund to be over 1.80m tall, estimate the probability of a man being over 1.80m tall. In addition, students are required to split into five groups and use an appropriate measuring instrument (tape) to conduct similar surveys for 200 people each, in five different locations and calculate the probability of over 1.80m tall in each case. Students should compare their results and also obtain a combined probability.
2. In a survey of Abuja households, 288 had home computers while 962 did not. Use this sample to estimate the probability of a household having a home computer.

3. An NPC survey of 600 people in the 18-25 age bracket found that 237 people smoke. If a person in that age bracket is randomly selected, find the approximate probability that he or she smokes.

Week 10

WEEK 11



4.7 Independent Events

The events E_1 *and* E_2 are independent if and only if $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$. In other words, the event E_1 *and* E_2 are independent if and only if the occurrence of event E_2 is not in any way affected by the occurrence of event E_1.

Independence

An event *B* is said to be independent of an event *A* if the probability that *B* occurs is not influenced by whether A has or has not occurred. In other words, if the probability of *B* equal the conditional probability of *B* given *A*.

$P(B) = P(B/A)$,

Now substituting $P(B)$ for $P(B/A)$ in the multiplication law;

$P(A \cap B) = P(A) \times P(B/A)$, we obtain;

$$P(A \cap B) = P(A) \times P(B)$$

We shall henceforth use the above equation as our formal definition of independence.

Example

A fair die and a coin are rolled at once, what is the probability of getting totals of head and an even number.

Solution

In this situation, the probability of obtaining an even number on the die and the probability of obtaining a head on the coin are independent. therefore, we obtain the joint probability of the two events as follows:

$P(H) = P(Head) = \frac{1}{2}$

$P(E) = P(Even\ number) = \frac{1}{2}$

$\therefore P(Head\ and\ even\ number) = P(H) \cdot P(E) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

An alternative method is to consider the entire sample space of the joint event as follows:

Alternative Solution

		Die					
		1	**2**	**3**	**4**	**5**	**6**
Coin	**H**	1H	2H	3H	4H	5H	6H
	T	1T	2T	3T	4T	5T	6T

Let the event for head and even number be denoted by:

$A = \{2H, 4H, 6H\}$

$\therefore P(Head\ and\ even\ number) = \frac{3}{12} = \frac{1}{4}$

4.8 Difference between Mutually Exclusive and Independent Events

The mutually exclusive events differ from the independent events in the sense that two events are mutually exclusive if their sets are disjoint regardless of whether the occurrences of one event influence the other. On the other hand, for the independent events, the occurrence of one of the events influences the occurrence of the other. The mutually exclusive events are better understood under the addition law of probability while the independent events through the multiplication law.

Definition

Event A and B are independent if and only if $P(A \cap B) = P(A) \times P(B)$ otherwise they are dependent.

Week 11

Theorem

If $E \text{ and } E'$ are complementary events, then $P(E') = 1 - P(E)$

Simple Proof

Since $E \text{ and } E'$ are complementary events, from set theory, $E \cup E' = S$ and also $E \text{ and } E'$ are disjoint.

Therefore;

$P(E \cup E') = P(S) = 1$

But $P(E \cup E') = P(E) + P(E')$

$\therefore P(E) + P(E') = 1$

$\therefore P(E') = 1 - P(E)$

Example

A coin is tossed six times in succession, what is the probability that at least one head occurs.

Solution

Week 11

There are $2^6 = 64$ sample points in the sample space since each toss can result in 2 outcomes an there are six coins. Note that an outcome may consist of at least one head or no head; hence the two events are complementary events.

$$P(at\ least\ one\ head) = 1 - P(\ no\ head)$$

$$P(at\ least\ one\ head) = 1 - \frac{1}{64} = \frac{63}{64}$$

Note that $P(\ no\ head) = \left[P(tail)\right]^6 = \frac{1}{2^6} = \frac{1}{64}$

4.9 Examples of Mutually Exclusive Events

Example

A bag contains 4 red, 5 yellow and 6 black identical balls. A ball is selected at random, what is the probability that the ball is yellow or black?

Solution

Week 11

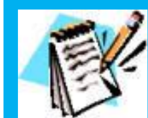
The events of a yellow or black ball are mutually exclusive because a ball can only belong either of the sets of yellow or black balls but not both.

$P(Yellow\ ball) = P(Y) = \frac{5}{15} = \frac{1}{3}$

$P(Black\ ball) = P(B) = \frac{6}{15} = \frac{2}{5}$

$\therefore P(Yellow\ or\ black\ ball) = P(Y) + P(B) = \frac{1}{3} + \frac{2}{5} = \frac{11}{15}$

Exercises



1) A die is loaded in such a way that an even number is twice as likely to occur as an odd number. If *A* is the event that a number less than 4 occurs on a single toss of the die, find *P(A)*.

2) In a random sample of 2000 men, 125 were fund to be over 1.78m tall, estimate the probability of a man being over 1.78m tall.

3) If a card is drawn from an ordinary deck of 52 cards, find the probability that it is a heart.

Week 11 Practical Activities

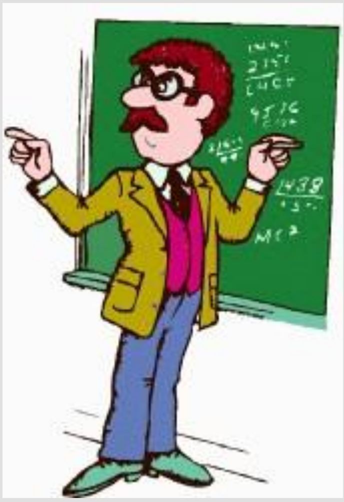
The Application of Probability to Medical Research

1. In a random sample of 100 children at a clinic, 35 were fund to be infected with malaria, estimate the probability of a child being infected with malaria. In

addition, students are required to split into five groups and visit five different clinics, in five different locations, to take samples of 150 children each and calculate the probability of malaria infection in each case. Students should compare their results and also obtain a combined probability for the five clinics.

2. When the allergy drug seldone was clinically tested, 70 people experienced drowsiness while 711 did not. Use this sample to estimate the probability of a seldone user becoming drowsy.

WEEK 12



5.1 Concepts of Probability

The mathematical theory of probability for finite sample spaces provides a set of number called weights, ranging from 0 to 1. To every point in the sample space be assign a weight such that the sum of all the weight is 1.

To find the probability of any event *A* we sum all weights assigned to the sample point in *A*. this sum is called the measure of *A* or the probability of *A* and is denoted by *P(A)*. Thus the measure of the set ϕ is zero and the measure of *S* is one.

Example 1

A coin is tossed twice, what is the probability that at least one head occurs.

Solution

The sample space for the experiment is $S = \{HH, HT, TH, TT\}$, if the coin is fair, each of these of these outcomes would be equally likely to occur. Therefore, we assign a weight of w to each sample point. Then $4w = 1$ *or* $w = \frac{1}{4}$. Let A represent the event of at least one head occurring, then;

$A = \{HH, HT, TH\}$

$\therefore P(A) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

Example 2

A die is loaded in such a way that an event number is twice as likely to occur as an odd number. What is the probability that event B of getting a 4 or a 5 when the die is tossed once?

Solution

Based on the sample space:

$S = \{1,2,3,4,5,6\}$

Let *w* be the weight of the odd numbers, then the weight of the event number is *2w*.

$P(S) = 1$

$\therefore w + 2w + w + 2w + w + 2w = 1$

$\therefore 9w = 1$

$\therefore w = \frac{1}{9}$

$\therefore P(odd\ number) = \frac{1}{9}$ and

$P(even\ number) = \frac{2}{9}$

now, we shall proceed as follows:

$\therefore P(a\ 4) = P(even\ number) = \frac{2}{9}$

$\therefore P(a\ 5) = P(odd\ number) = \frac{1}{9}$

$\therefore P(a\ 4\ or\ a\ 5) = P(a\ 4) + P(a\ 5) = \frac{2}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$

Example

Let *A* and *B* be events find an expression and exhibit the vein diagram for the event that:

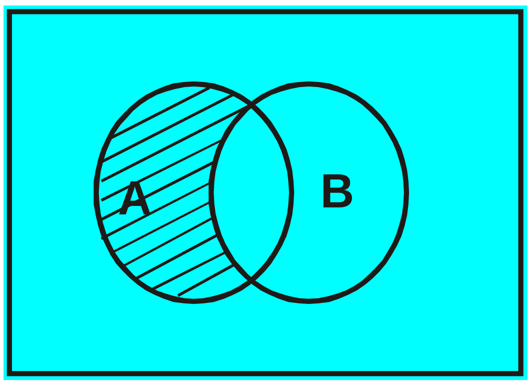
i) *A* but not *B* occur, i.e. only *A* occurs

ii) Either *A* or *B*, but not both occurs i.e. exactly one of the two events occurs.

Solution

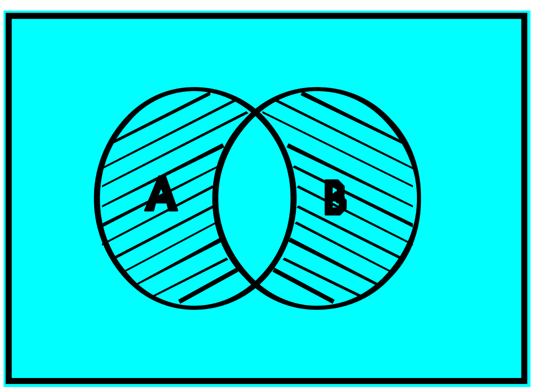
Week 12

i) Since *A* but not *B* occurs, shade the area *A* outside *B*. In other words, the given event is $A \cap B'$ as shown below:



A but not B occurs

ii) Since *A* or *B* but not both occur shade the area of *A* and *B* except where they intersect. Thus the given event is $(A \cap B') \cup (B \cap A')$ as shown below:



Either A or B but not both occurs

5.2 Probability as a Function of the Sample Space

This is often the experimental approach to probability. To find the probability of any event *A* we sum all weights assigned to the sample point in *A*. this sum is called the measure of *A* or the probability of *A* and is denoted by *P(A).*

Example 1

A coin is tossed twice, what is the probability that at least one head occurs.

Solution

The sample space for the experiment is $S = \{HH, HT, TH, TT\}$, if the coin is fair, each of these of these outcomes would be equally likely to occur. Therefore, we assign a weight of *w* to each sample point. Then $4w = 1 \; or \; w = \frac{1}{4}$. Let *A* represent the event of at least one head occurring, then;

$$A = \{HH, HT, TH\}$$

$$\therefore P(A) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

Example 2

A die is loaded in such a way that an event number is twice as likely to occur as an odd number. What is the probability that event *B* of getting a 4 or a 5 when the die is tossed once?

Solution

Based on the sample space:

$S = \{1,2,3,4,5,6\}$

Let *w* be the weight of the odd numbers, then the weight of the event number is *2w*.

$P(S) = 1$

$\therefore w + 2w + w + 2w + w + 2w = 1$

$\therefore 9w = 1$

$\therefore w = \frac{1}{9}$

$\therefore P(odd\ number) = \frac{1}{9}$ and

$P(even\ number) = \frac{2}{9}$

now, we shall proceed as follows:

$\therefore P(a\ 4) = P(even\ number) = \frac{2}{9}$

Week 12

$\therefore P(a\ 5) = P(odd\ \ number) = \frac{1}{9}$

$\therefore P(a\ 4\ or\ \ a\ 5) = P(a\ 4) + P(a\ 5) = \frac{2}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$

5.3 Relative Frequency Approach to Probability

The relative frequency approach to probability is simply the empirical approach to probability. The probability, often called the 'relative frequency' is obtained by dividing each frequency by the total frequency.

Example

The following table shows the distribution of various vehicles that exit a toll plaza.

Vehicle	Number of Vehicle
Cars	25
Trucks	15
Lorries	30
Bikes	10
Others	20
Total	100

What is the probability that a vehicle about to pass the toll plaza will be car?

Solution

Using the relative frequency approach; the probability of a car is calculated as follows:

$$P = P(Car) = \frac{f_i}{\sum f_i} = \frac{25}{100} = 0.25$$

5.4 Axiomatic Approach of Probability

Historically, probability theory began with a study of games of chance, such as die and card. The probability *P* of an event *A* was defined as follows:

If *A* can occur in *S* ways out of a total on *n* equally likely ways, then:

$$P = P(A) = \frac{s}{n}$$

For example, in tossing a die an even number can occur in 3 ways of 6 "equally likely" ways, hence $P = \frac{3}{6} = \frac{1}{2}$. This classical definition of probability is essentially circular since the idea of "equally likely" is the same as that of "with equal probability" which has not been defined. The modern treatment of probability theory is purely axiomatic. This means that the probabilities of our events can be perfectly arbitrary, except that they must satisfy certain axioms listed below. The classical theory will correspond to the special case of so-called equiprobable spaces.

Week 12 Practical Activities

Week 12

The Application of Probability in Health Care Delivery

1. The following table shows the distribution of various common child killer diseases reported in a rural clinic in Nigeria.

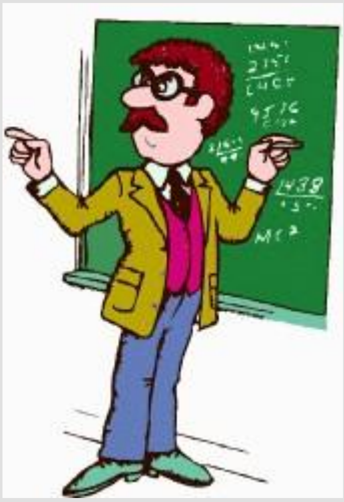
Type of Disease	Number of Infected Children
Measles	25
Whooping Cough	15
Malaria	30
Diarrhoea	20
Others	10
Total	**100**

What is the probability that an infected child will suffer from whooping cough? Students are required to split into two groups with one group visiting a rural clinic and the other visiting an urban clinic to collect data for 200 children each. The data should be tabulated in the above format and probabilities calculated for each type of disease. The results for the two groups should be compared.

2. Data provided by the police headquarters in Nigeria revealed that for a representative sample of convicted burglars, 76,000 were jailed, 25,000 were put on probation, and 2,000 received other sentences. Use these results to estimate the probability that a convicted burglar will serve jail term.

Week 12

WEEK 13



5.5 Calculations of Probability

If a die is tossed in the air, then it is certain that the die will come down, but it is not certain that, say, a 6 will appear. However, suppose we repeat this experiment of tossing a die, let *s* be the number of successes, i.e. the number of times a 6 appears, and let *n* be the number of tosses. Then it has been empirically observed that the ratio $f = \frac{s}{n}$ called a relative frequency, become stable in the long run i.e. approached a limit. This stability is the basis of probability theory.

Definition

The probability of any event *A* is the sum of the weights of all sample points in *A*. Therefore:

$$0 \le P(A) \le 1, \quad P(\Phi) = 0, \quad P(S) = 1$$

In probability theory, we define a mathematical model of the above phenomenon by assigning “probabilities” (or: the limit values of the relative frequencies) to the “events” connected with an experiment. Naturally, the reliability of our mathematical model for a given experiments depends upon the closeness of the assigned probabilities to the actual relative frequency. This then gives rise to problems of testing and reliability which form the subject matter of statistics.

Historically, probability theory began with a study of games of chance, such as die and card. The probability *P* of an event *A* was defined as follows:
If *A* can occur in *S* ways out of a total on *n* equally likely ways, then:

$$P = P(A) = \frac{s}{n}$$

For example, in tossing a die an even number can occur in 3 ways of 6 “equally likely” ways, hence $P = \frac{3}{6} = \frac{1}{2}$. This classical definition of probability is essentially circular since the idea of “equally likely” is the same as that of “with equal probability” which has not been defined. The modern treatment of probability theory is purely axiomatic. This means that the probabilities of our events can be perfectly arbitrary, except that they must satisfy certain axioms listed below. The classical theory will correspond to the special case of so-called equiprobable spaces.

Example

A bag contains 4 red, 5 yellow and 6 black identical balls. A ball is selected at random, what is the probability that the ball is yellow or black?

Week 13

Solution

The events of a yellow or black ball are mutually exclusive because a ball can only belong either of the sets of yellow or black balls but not both.

$P(Yellow\ ball) = P(Y) = \frac{5}{15} = \frac{1}{3}$

$P(Black\ ball) = P(B) = \frac{6}{15} = \frac{2}{5}$

$\therefore P(Yellow\ or\ black\ ball) = P(Y) + P(B) = \frac{1}{3} + \frac{2}{5} = \frac{11}{15}$

5.5 Properties of Probability

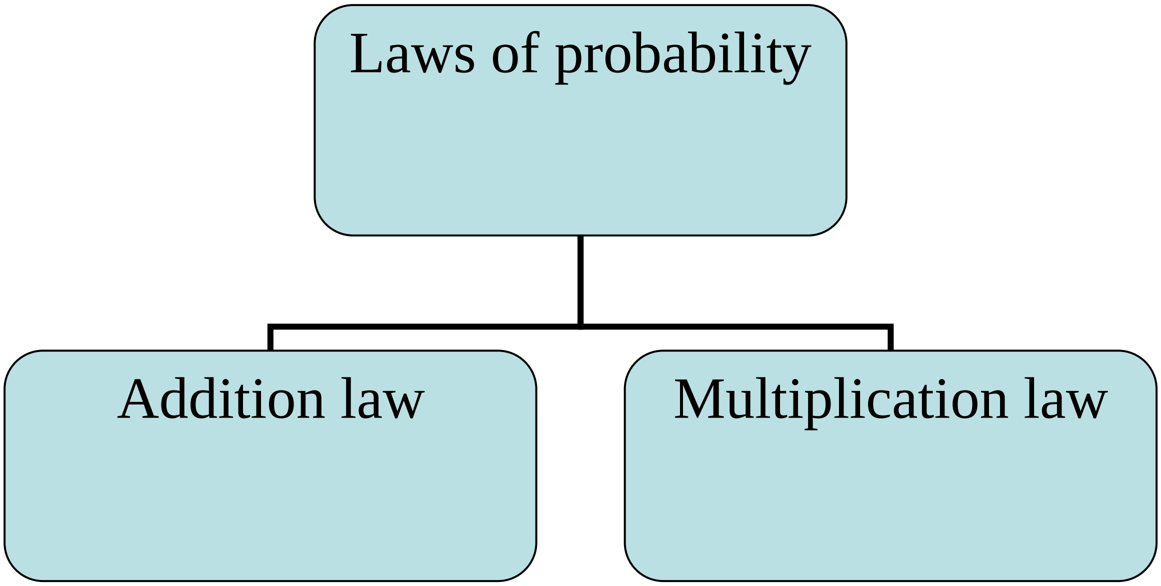
The probability of any event *A* is the sum of the weights of all sample points in *A*. Therefore:

1. $0 \le P(A) \le 1$
2. $P(\phi) = 0$
3. $P(S) = 1$

In probability theory, we define a mathematical model of the above phenomenon by assigning "probabilities" (or: the limit values of the relative frequencies) to the "events" connected with an experiment. To every point in the sample space be assign a weight such that the sum of all the weight is 1.

To find the probability of any event *A* we sum all weights assigned to the sample point in *A*. This sum is called the measure of *A* or the probability of *A* and is denoted by *P(A)*.

Week 13



5.6 Addition Law of Probability

The addition law of probability is used when we are interested in the probability of one event or the other. For two events E_1 *and* E_2, the probability of E_1 *or* E_2 is symbolically written as follows:

$$P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$$

There are two cases under the addition law are discussed as follows:

Case I: Non-Mutually Exclusive Events

If E_1 *and* E_2 are non-mutually exclusive events, then:

$$P(E_1 \text{ or } E_2) = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

Case II: Mutually Exclusive Events

If E_1 *and* E_2 are mutually exclusive events, then:

$$P(E_1 \text{ or } E_2) = P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

Generalizing this case; if $E_1, E_2, ..., E_n$ are mutually exclusive events then:

$$P(E_1 \cup E_2 \cup E_3 \cup .... \cup E_n) = P(E_1) + P(E_2) + P(E_3) + ... + P(E_n)$$

Week 13

Note

If $E_1, E_2, ..., E_n$ is a partition of a sample space S, then;

$$P(E_1 \cup E_2 \cup E_3 \cup .... \cup E_n) = P(E_1) + P(E_2) + P(E_3) + ... + P(E_n) = P(S) = 1$$

Example

A bag contains 4 red, 5 yellow and 6 black identical balls. A ball is selected at random, what is the probability that the ball is yellow or black?

Solution

The events of a yellow or black ball are mutually exclusive because a ball can only belong either of the sets of yellow or black balls but not both.

$$P(Yellow\ ball) = P(Y) = \tfrac{5}{15} = \tfrac{1}{3}$$

$$P(Black\ ball) = P(B) = \tfrac{6}{15} = \tfrac{2}{5}$$

$$\therefore P(Yellow\ or\ black\ ball) = P(Y) + P(B) = \tfrac{1}{3} + \tfrac{2}{5} = \tfrac{11}{15}$$

Week 13

5.6 Multiplication Law of Probability

This states that the probability of a combined occurrence of two (or more) events E_1 *and* E_2 is the product of the probability of E_1 and the conditional probability of E_2 on the assumption that E_1 has occurred. This is denoted by $P(E_1 \text{ and } E_2)$ or simply

$P(E_1 \cap E_2) = P(E_1) \times P(E_2 / E_1)$

Where:

$P(E_2 / E_1)$ is the conditional probability of event E_2 on the assumption that E_1 occurs at the same time. Generalizing the above law; if in an experiment the events.

$E_1, E_2, ..., E_n$ can occur then:

$$P(E_1 \cap E_2 \cap E_3 \cap .... \cap E_n) = P(E_1) \times P(E_2 / E_1) \times P(E_3 / E_1 E_2) \times .... \times P(E_n / E_1 E_2 ... E_n)$$

Multiplication Law for Independent Events

The multiplication law becomes simpler when the events E_1 *and* E_2 are independent where;

$$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$$

I.e. for independent events $P(E_2 / E_1) = P(E_2)$

Also, generalizing for the independent events; if in an experiment the events $E_1, E_2, ..., E_n$ can occur then:

$$P(E_1 \cap E_2 \cap E_3 \cap .... \cap E_n) = P(E_1) \times P(E_2) \times P(E_3) \times .... \times P(E_n)$$

Week 13 Practical Activities

Week 13

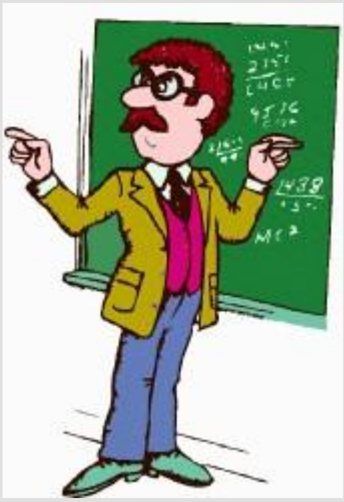
Application of Probability to Research

1. Assuming that political affiliations are mutually exclusive, students are required to split into five groups with each group assigned to a particular neighbourhood and collect data on the political affiliations of 100 registered adults in the neighbourhoods. Hence, calculated the probability that a randomly selected voter will be affiliated to a particular party. Moreover, using the addition laws of probability, obtain the coalition between different political parties at the poll.
2. Blood groups are determined for a sample of people and the results are given in the following table. If one person from this sample is randomly selected, find the probability that the person has group AB blood.

Blood group	frequency
O	90
A	80
B	20
AB	10

Week 13

WEEK 14



5.9 Conditional Probability

In many problems and situations, however, the events are neither independent nor mutually exclusive, and the general theory of conditional probability will have to be applied.

Definition

Let E be an arbitrary event in a sample space S with $P(E) > 0$. The probability that an event A occurs once E has occurred or, in other words, the ***conditional probability*** of a given E, written

$P(A/E)$ is defined as follows:

$$P(A/E) = \frac{P(A \cap E)}{P(E)}$$

It is obvious that, $P(A/E)$ in a certain sense measures the relative probability of A with respect to the reduced space E.

Theorem

Let S be a finite equiprobable space with events A and E. Then:

$$P(A/E) = \frac{P(A \cap E)}{P(E)} = \frac{Number\ of\ elements\ in\ A \cap E}{Number\ of\ elements\ in\ E}$$

Or

$$P(A/E) = \frac{Number\ of\ ways\ \ A\ and\ E\ can\ occur}{Number\ of\ ways\ E\ can\ occur}$$

Example 1

Let a pair of fair dice be tossed. If the sum is 6, find the probability that one of the dice is a 2.

Solution

Let the event for sum of six be E

And let the event for a die shows of 2 be A

Then we are interested in $P(A/E) = \frac{P(A \cap E)}{P(E)}$

Consider the sample space as follows:

S	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	***1,5***	1,6
2	2,1	2,2	2,3	***2,4***	2,5	2,6
3	3,1	3,2	***3,3***	3,4	3,5	3,6
4	4,1	***4,2***	4,3	4,4	4,5	4,6
5	***5,1***	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

$$P(E) = \frac{5}{36}$$

$$P(A \cap E) = \frac{2}{36}$$

$$\therefore P(A/E) = \frac{P(A \cap E)}{P(E)} = \frac{2/36}{5/36} = \frac{2}{5}$$

Example 2

A lot contains 12 items of which 4 are defective. Three items are drawn at random from the lot one after the other without replacement. Find the probability *P* that all the three are non-defective.

Solution

The probability that the first item is non- defective is 8/12 since 8 of 12 items are non-defective. If the first item is non-defective, then the probability that the next item is non-defective is 7/11 since only 7 of the remaining 11 items are non-defective. If the first two items are non-defective is 6/10 since only 6 of the remaining 10 items are now non-defective. Thus by the multiplication theorem:

$$P = \frac{8}{12} \cdot \frac{7}{11} \cdot \frac{6}{10} = \frac{14}{55}$$

The Concept of Independence

An event *B* is said to be independent of an event *A* if the probability that *B* occurs is not influenced by whether A has or has not occurred. In other words, if the probability of *B* equal the conditional probability of *B* given *A*.

$P(B) = P(B/A)$,

Now substituting $P(B)$ for $P(B/A)$ in the multiplication law;

$P(A \cap B) = P(A) \times P(B/A)$, we obtain;

$$P(A \cap B) = P(A) \times P(B)$$

We shall henceforth use the above equation as our formal definition of independence.

Definition

Event *A* and *B* are independent if and only if $P(A \cap B) = P(A) \times P(B)$ otherwise they are dependent.

Week 14

Theorem

If $E \text{ and } E'$ are complementary events, then $P(E') = 1 - P(E)$

Proof

Since $E \text{ and } E'$ are complementary events, from set theory, $E \cup E' = S$ and also $E \text{ and } E'$ are disjoint.

Therefore;

$P(E \cup E') = P(S) = 1$

But $P(E \cup E') = P(E) + P(E')$

$\therefore P(E) + P(E') = 1$

$\therefore P(E') = 1 - P(E)$

Example

A coin is tossed six times in succession, what is the probability that at least one head occurs.

Solution

Week 14

There are $2^6 = 64$ sample points in the sample space since each toss can result in 2 outcomes an there are six coins. Note that an outcome may consist of at least one head or no head; hence the two events are complementary events.

$$P(at\ least\ one\ head) = 1 - P(\ no\ head)$$

$$P(at\ least\ one\ head) = 1 - \frac{1}{64} = \frac{63}{64}$$

Note that $P(\ no\ head) = [P(tail)]^6 = \frac{1}{2^6} = \frac{1}{64}$

Exercise

1) The probability that Mr. Bala will be alive in 25 years time s 0.6 and the probability that Mr. Audu will be alive in 25 years is 0.9. What is the probability that neither will be alive in 25 years?

2) The probably that a married man watches a certain television show is 0.4 and the probability that a married woman watches the show is 0.5. The probability that a man watches the show given that his wife does is 0.7. Calculate;
   (a) The probability that a couple watches the show
   (b) The probability that a wife watches the show given that her husband does
   (c) The probability that at least one person of a married couple will watch the show.

3) What is the probability of getting a total of 7 or 11 when a pair of dice is tossed.

4) The National library carries 50 Nigerian magazines that focus on either news or sports. Thirty of them focus on news and the remaining 20 focus on sports.

Among the 30 news magazines, 20 include international news and 10 include national news. Among the 20 sports magazines, 5 focus on international sports and the remaining 15 focus on national sports.

a. Create on table illustrating these numbers with type of magazine (news, sports) as the row variable.

b. What proportion of the magazine include international sports?

5.10 Bayes Theorem

Theorem

Suppose $A_1, A_2, \ldots, A_n$ is a partition of *S* and also *B* is any amount. Then for any *i*

$$P(A_i/B) = \frac{P(A_i)P(B/A_i)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + \ldots + P(A_n)P(B/A_n)}$$

Note

The Bayes theorem is an extention of conditional probability where the condition is a partitioned set.

Week 14 Practical Activities

Week 14

Application of Conditional Probability

Two psychologists surveyed 478 children in senior secondary 1, 2 and 3 in some public secondary schools, in Nigeria. They stratified their sample, drawing roughly equal number of students from rural, suburban and urban secondary schools. Among other questions, they asked the students whether their primary goal was to get good grades, to be popular, or to be good at sports. One question of interest was whether boys and girls at this age had similar goals. Here is a contingency table giving counts of the students by their goals and gender.

Gender	**Goals**			**Total**
	Grades	Popular	Sports	
Boys	117	50	60	**227**
Girls	130	91	30	**251**
Total	**247**	**141**	**90**	**478**

Here the sample space is a set of 478 students. If we select a student at random from this study, the probability that we select a girl is just the corresponding relative frequency (since we are equally likely to select any of the 478 students). There are 251 girls in the data out of a total of 478, giving a probability of:

$$P(girl) = \frac{251}{478} = 0.525.$$

The same method works for more complicated events like intersections. For example, what is the probability of selecting a girl whose goal is to be popular? Well, 91 girls named popularity as their goal, so the probability is:

$$P(girl\ and\ popular) = \frac{91}{478} = 0.190$$

The concept of conditional can better be understood from the table. What if we are given the information that the selected student is a girl? Would that change the probability that the selected student's goal is sports? Of course, it would. When we restrict our focus to girls, we look only at the girls' row of the table, which gives the conditional probability of goals given girl. Out of the 251 girls, only 30 of them said their goal was to excel at sports.

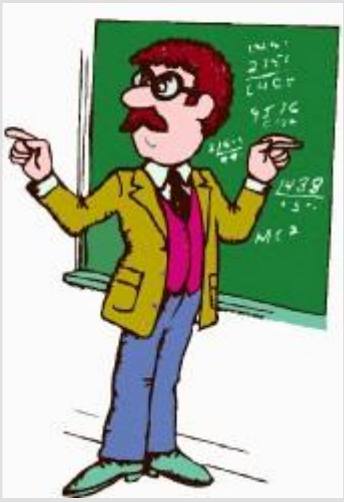
We write the probability that a selected student wants to excel at sports given that we have selected a girl is:

$$P(sports / girl) = \frac{30}{251} = 0.120$$

Having demonstrated this, students are required to split into four groups with each group visiting a particular secondary school and collect a similar data for at least 300 students. Hence, using the collected data, students are required to calculate similar probabilities.

Week 14

WEEK 15



5.10 The Use of Bayes Theorem

Suppose that events $A_1, A_2, ..., A_n$ form a partition of a sample space *S*, that is events A_i are mutually exclusive and their union is *S*. Now let *B* be any other event. Then;

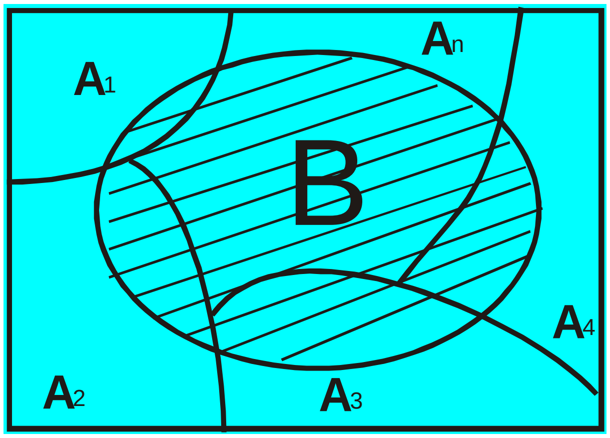
$$B = S \cap B = (A_1 \bigcup A_2 \bigcup .... \bigcup A_n) \bigcap B$$

$$B = S \cap B = (A_1 \bigcap B) \bigcup (A_2 \bigcap B) \bigcup .... \bigcup (A_n \bigcap B)$$

Where the $(A_i \bigcup B)$ are also mutually exclusive. Accordingly;

$$P(B) = P(A_1 \bigcap B) + P(A_2 \bigcap B) + ... + P(A_n \bigcap B)$$

This is diagrammatical shown as follows:



Thus by multiplication theorem;

$$P(B) = P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + ... + P(A_n)P(B/A_n)$$

On the other hand, for any *i*, the conditional probability of A_i *given* B is defined by the following:

$$P(A_i / B) = \frac{P(A_i \cap B)}{P(B)}$$

If we substitute $P(B) = P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + ... + P(A_n)P(B/A_n)$ and also substitute $P(A_i \cap B) = P(A_i)P(B/A_i)$ we obtain the following theorem.

Theorem

Suppose $A_1, A_2, ..., A_n$ is a partition of *S* and also *B* is any amount. Then for any *i*

$$P(A_i / B) = \frac{P(A_i)P(B/A_i)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + ... + P(A_n)P(B/A_n)}$$

Example

Three machines A, B, and C produce respectively 50%, 40% and 20% of the total number of items of a factory. The percentages of defective output of these machines are 3%, 4% and 5%. If an item is selected at random, find the probability that the item is defective.

Solution

Let X be the event that an item is defective. Then by Bayes theorem's partition

$$P(X) = P(A)P(X/A) + P(B)P(X/B) + P(C)P(X/C)$$

$$P(X) = (0.50)(0.03) + (.030)(0.04) + (0.20)(0.05) = 0.37$$

Example

Consider the factory I the proceeding example. Suppose an item is selected at random and found to be defective: Find the probability that the item was produced by machine A;

that is, find $P(A/X)$.

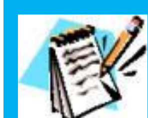
Solution

$$P(A/X) = \frac{P(A)P(X/A)}{P(A)P(X/A) + P(B)P(X/B) + P(C)P(X/C)}$$

$$\therefore P(A/X) = \frac{(0.50)(0.03)}{(0.50)(0.03) + (0.30)(0.04) + (0.20)(0.05)} = \frac{15}{37}$$

In other words, we divide the probability of the required path by the probability of the reduced sample space i.e. those paths which lead to a defective item.

Exercise



1) Three members of a recreational club in Nigeria have been nominated for the office of the president. The probability that Mr. Adams will be elected is 0.3, the probability that Mr. Isa will be elected is 0.2. Should Mr. Adams be elected; the probability for an increase in membership fees is 0.8. Should Mr. Benson or Mr. Isa be elected, the corresponding probabilities for an increase in fees are 0.2 and 0.4 respectively. If someone is considering joining the club but delays his decision for several weeks only to find out that the fees have been increased, what is the probability that Mr. Isa was elected the president of the club? (Answer: $\frac{8}{37} = 0.22$)

2) A commuter owns two cars, one a compact and the other a standard model. About 75% of the time he uses the compact to travel to work and about 25% of

the time the larger car is used. When he uses the compact car, he usually gets home by 5:30 pm about 75% of the time. If he uses the standard size car, he gets home by 5:30pm about 60% of the time. If he gets home at 5:35pn, what is the probability that he used the compact car?

Week 15 Practical Activities

Industrial Application of Bayes Theorem

A TV set contains five circuit board of type A, five of type B and three of type C. the probability of failing in its first 5000 hours of use is 0.01 for type A circuit board, 0.02 for type B circuit board and 0.025 for a type C circuit board. Assuming that the failures of the various circuit boards are independent of one another, compute the probability that no circuit board fails in the first 5000 hours of use.

Having had solve this problem, students are required to visit any industrial setting to obtain similar data and calculate probabilities using Bayes theorem.